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REVIEW OF THE INTERACTION BETWEEN THE GEOMAGNETIC FIELD AND THE SOLAR CORPUSCULAR RADIATION

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R. Blum

M. L. Report No. 975

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ACKNOWLEDGEMENTS

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INTRODUCTION

There is at present a rapidly expanding body of empirical data and theoretical investigations to substantiate the concept of of the interaction between the geomagnetic field (GMF) and the solar corpuscular radiation (SCR) and its concomitant effects. The evidence for some sort of interaction between the geomagnetic field and the tenuous, highly ionized plasma issuing from the sun is indisputable, but the precise nature of this interaction is still an open question. This is due partly to the inherent complexity of the theoretical problem, and partly to the lack of sufficient experimental data. However, it can be said that the approximate theory which does exist agrees generally with the direct measurements which have been made by rocket probes.

The general outline of this theory is that the solar corona is continually expanding outward through the solar system in the form of a very low-density fully-ionized plasma consisting of protons and free electrons. This plasma is variously known as the interplanetary gas, the solar wind, and the solar corpuscular radiation (SCR); its density near the earth is estimated to be around 10 protons/cm^3 , drift velocity 300 km/sec , and thermal kinetic energies between 10^5 $^\circ K$ and 10^6 $^\circ K$. It is generally believed, without any notable dissent as of this writing, that the GMF is almost completely separated from the SCR by an interface, or transition region; theoretical estimates yield a minimum thickness of the order of a proton Larmor radius in the GMF -- about one kilometer. In fact, Grad, (1961), has shown this condition to obtain generally for transition regions between magnetic fields and plasmas, where frozen-flow dominates over the diffusion of plasma across field lines. This concept of the interface will be qualified somewhat as we proceed, but remains essentially valid.

The impact of the SCR compresses the GMF on the day side, and elongates it on the night side of the earth. Since the thermal pressure, nkT, of the SCR is less than one-tenth of its dynamic pressure $\frac{1}{2}$ nmv^2 , the shape of the GMF-SCR interface is largely independent of the temperature of the

stream on the day side. However, as one moves away from the sun the GMF decreases and the thermal pressure in the SCR should eventually close the interface, so the field lines do not extend to infinity.

The interface may thus be considered as a mathematical surface in a free-boundary problem separating field on one side of the boundary from plasma on the other. The boundary conditions then require surface currents, J, in the interface which must exactly cancel the (distorted) GMF outside of the surface and augment it inside the surface, in the region known as the magnetosphere. If there is to be no field outside the magnetosphere then the total field must be everywhere tangential to the surface.

There is good reason to believe that the size and the shape of the interface is largely determined by a balance of forces due to $\underline{J} \times \underline{H}$ force on the surface currents, and the change of momentum of plasma particles reflected from the interface. The situation is depicted schematically in Fig. 1; both theory and measurement agree on $r_0 \doteq 10$ R (R = earth radius) .

In addition to this steady-state effect there is also a definite correlation between solar activity, geomagnetic storms, and auroral activity. Particles from a solar flare may average speeds of 1000 km/sec or more, with increased densities, depending upon the strength of the flare (IGY, 1962a); the increased flux density perturbs and compresses the GMF giving rise to the increased fluctuations in the GMF which are measured at the earth's surface. Furthermore, the perturbation of field lines may cause "dumping" of trapped particles from the radiation belts into the auroral zones, which gives rise to increased auroral displays during geomagnetic storms (IGY, 1961a).

In this review our emphasis will be primarily on the theoretical aspects of the problem, especially the quantitative, though idealized, theory which has been worked out in detail. However, there is also a large body of semi-quantitative and qualitative theory to which we shall refer, although not with the same attention to mathematical details. Since it is impossible to review all the material pertaining to this subject, our aim is to make this review self-contained by presenting enough material on each aspect to provide a physical basis and theoretical framework for understanding.

SOLAR
WIND

NEUTRAL
POINT

INTERFACE

EARTH

DIPOLE

r_0

MAGNETIC
FIELD LINES
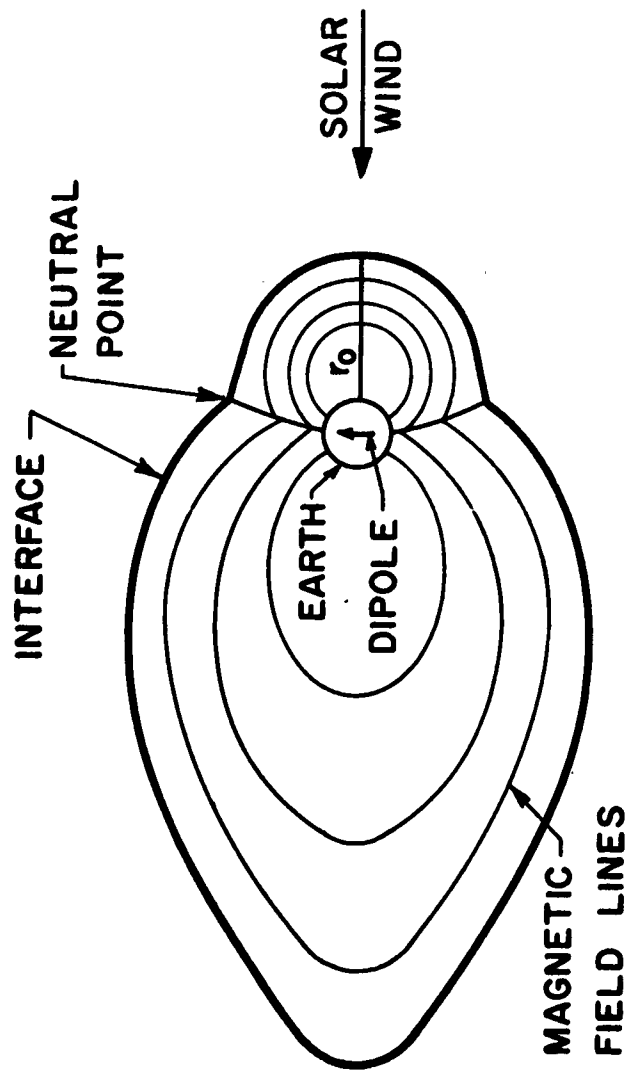
IDEALIZED MODEL OF THE GEOMAGNETIC CAVITY IN
THE SOLAR WIND, NOON MERIDIAN PLANE

FIG. 1--Idealized model of the geomagnetic cavity in the SCR, noon meridian plane.

Our review will be organized as follows:

I. EXPERIMENTAL EVIDENCE

A. THE SOLAR CORONA

The sun is divided into three regions: (1) the photosphere, which is seen with the naked eye; (2) the chromosphere, visible as a rosy arc for a few seconds during an eclipse; and (3) the corona, seen as a pearly halo surrounding the lunar disk during a total eclipse. The photosphere is the body of the sun, with radius $R\odot$ = 700,000 km ; the corona is the solar atmosphere, and the chromosphere forms a relatively thin ($\sim$40,000 km) transition region between them (Aller, 1953). By considering the sun to be a blackbody radiator and by examining the energy distribution of its radiation, the temperature at its surface is found to be about 6000 $^{\circ}K$ .. As one moves radially outward through the chromosphere the temperature increases to 1,000,000 - 2,000,000 $^{\circ}K$ while the density drops from 10^{16} to 10^{8} atoms/cm^{3} at the base of the corona.

The luminous corona is highly irregular in shape, and approximately one to two solar radii in width; however, radial streamers are conspicuous, and one as long as 15R$\odot$ (Chapman, 1961c) has been recorded. It may be more useful to think of the corona as continually expanding outwards through the solar system, and thus constituting the SCR as well. The luminous corona consists of three components: K, E, and F: the K-component has a continuous spectrum with no absorption lines, and is due to scattering by free electrons; thus its light reaches us partly polarized; and from measurements of polarization and luminosity the free electron densities are calculated (Van de Hulst, 1953; Chapman 1961c).

The E-component has a pure emission spectrum of bright lines; temperatures determined from thermal Doppler widths are about 2 million $^{\circ}K$; whereas the degree of ionization determined from the emission line intensities indicates a temperature of 10^{6} $^{\circ}K$. Other determinations also yield temperatures in this range. The differences have not yet been resolved, and beyond 3R$\odot$ the temperatures can only be inferred indirectly from electron density data based upon eclipse observations (Chapman, 1961c).

Between $3R\Theta$ and $20R\Theta$ the temperature drops from 10^6 $^\circ K$ to 3×10^5 $^\circ K$; a fact which leads Parker (see II.B) to assume the corona to be approximately isothermal in this region.

The F-component is due to scattering from interplanetary dust between the sun and the earth.

B. THE SOLAR CORPUSCULAR RADIATION

Biermann (1951) demonstrated that radiation pressure alone is insufficient to account for the observed deflections of comet tails passing near the sun. These experienced extreme outward accelerations which he believed due to the emission of a neutral stream of protons and electrons which would yield densities and velocities of around 100 protons/cm^3 and 500 km/sec at 1 a.u. Kiepenheuer (1953) has summarized Biermann's determinations of the SCR and the work has been extended by Biermann (1952, 1953, 1957, 1960, 1961). In the case of a solar flare, estimated values were as high as 10^4 protons/cm^3 and 1500 km/sec.

The Explorer X rocket probe, launched in March, 1961, measured quiescent values of approximately 6-10 protons/cm^3 and 300 km/sec (IGY Bull. 1962a; Bridge, et. al., 1961). During the experiment a solar flare caused a geomagnetic storm at the earth; this appeared to Explorer X as a sudden rise in flux density and particle energies beyond $37R$ from the geocenter. Preliminary results indicate velocities as high as 700 km/sec in the SCR associated with the flare.

Explorer X measurements further indicate that the "temperature" of the SCR is between 10^5 and 10^6 $^\circ K$ (Bonetti et al, 1962; Rossi, 1962). According to Bonetti, et. al. (1962) the lower temperature is suggested by variations in particle directions as measured by Explorer X; the upper temperature, by the spread in particle energies. They conclude: "These results are rather crude but they indicate that the motion of the particles is not isotropic. Indeed it seems that the thermal motions are predominantly confined to directions parallel to the magnetic field in the frame of reference moving with the bulk velocity."

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C. THE HELIOMAGNETIC FIELD (HMF)

Until the Babcocks' invention of the solar magnetograph in 1952, the HMF of the undisturbed sun was too small for detection; since then the sun has been observed to have a dipole-like field of approximately 1 gauss at its poles, and of opposite polarity (Babcock H.W. and Babcock, H.D., 1955). However, towards the solar equator the field becomes rather disordered, although still approximately one gauss.

The streamers in the corona suggest that the HMF is radial; this is confirmed by the observations of Hewish (1958). He observed the scattering of radio signals from the Crab nebula as it moved past the sun, and found the HMF to be radial out to $30R_\odot$, which was the limit of his observations. Furthermore, the results of the Pioneer I (Sonett, et. al., 1960a), Pioneer V (Coleman et. al., 1960; IGY Bull., 1960), and Explorer X (IGY Bull., 1962a; Bridge, et. al., 1961) space probes indicate the presence of an interplanetary field of approximately $2 - 10\gamma$ ($1\gamma = 10^{-5}$ gauss). Fields of the order of $6 - 12\gamma$ were measured by Pioneer I and Explorer X just outside the magnetosphere, and Pioneer V measured fields of the order of $1 - 7\gamma$ at distances of five million km from the earth, as well as extreme values of 14γ and over, which were measured simultaneously with the onset of geomagnetic storms at the earth. Both phenomena were correlated with the appearance of solar flares. Pioneer V measurements also indicate the presence of a steady HMF component of about 2.7γ, a figure in agreement with Parker's theory (see II.B) that the HMF decreases as $1/r^2$ from one gauss at the surface of the photosphere.

The study of rise and decay times of cosmic-ray activity at sea level due to solar flares substantiates the notion that charged particles leaving the sun follow curved trajectories along magnetic lines of force originating in the sun, and do not come directly to the earth. These field lines are thought to stretch outward from the sun with a characteristic spiral shape due to the solar rotation, suggestive of water jets from a rotating lawn sprinkler (Fig. 2). Hence the name "garden hose effect." The persistence

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HMF LINES

SUN

TRAJECTORY
OF SOLAR FLARE
PARTICLES ALONG
HMF LINE

50°

EARTH

EAST ◄——●——► WEST
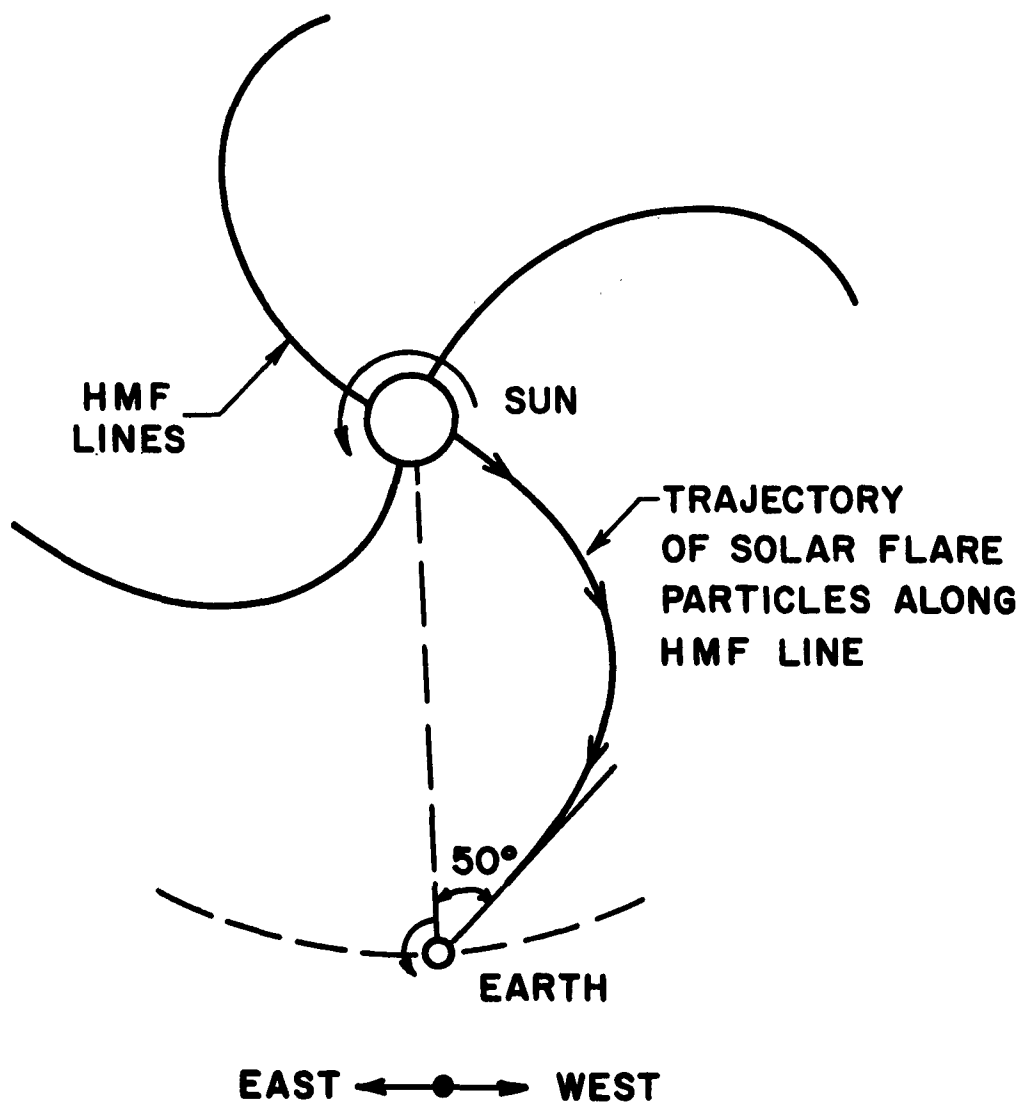
FIG. 2--HMF and effect on solar cosmic radiation.

of solar cosmic[1] activity long after the flare has disappeared indicates that the rays may be trapped within the HMF, which in turn is stretched outward due to the pressure of the trapped particles.

McCracken, (1962), has described the cosmic-ray flare effects and properties of the HMF which may be deduced from them. He notes that the majority of events are due to solar flares on the west limb of the sun. Although some events are caused by flares near the center of the solar disk, and in one case by a flare in the eastern 40% of the disk, rise times of solar cosmic ray activity measured at sea level (about 30 minutes) are least for flares occurring near the western limb. This argues that particles diffuse across HMF lines as they travel toward the earth, so that particles from the western limb travel the most direct HMF line from sun to earth, whereas only the more diffuse edge of a cloud of particles from the central portion of the sun actually reaches the earth. Furthermore, McCracken notes that very energetic particles generally arrive from directions making angles of around 50° with the earth-sun line; this agrees with the direction of the HMF as measured by Explorer X.

The Forbush decrease is noted just after the arrival of the solar cosmic rays: this is a decrease in the amount of galactic cosmic radiation, as distinct from solar cosmic radiation, which increases. Pioneer V has observed a Forbush decrease of about the same intensity as a Forbush decrease measured simultaneously at the earth. At the same time the average HMF increased from 5 to 15 gammas and later to 40 gammas, finally returning to 2 gammas (ambient value) within 48 hours of onset. This lends support to the theory that the HMF is compressed and blown outward by the solar flare in the form of a magnetohydrodynamic shock wave,and that the increased field screens out the normal galactic cosmic rays. These measurements also indicate that while the first phase of a geomagnetic storm is dependent upon solar activity, the main phase is strictly a geophysical effect whose cause lies within the magnetosphere,

[1]Cosmic Rays - highly penetrating corpuscular radiation originating outside the magnetosphere. Solar cosmic rays originate in the sun; galactic rays originate outside of the solar system. Cosmic Rays consist almost entirely of positive ions, two-thirds of which are protons.

since the main phase generally persists much longer than 48 hours.

The fact that the solar cosmic radiation continues to be detected for many hours and even days after onset shows that the increased fields also keep the solar radiation from moving outward. Parker (1959) has suggested that this may also be due to the presence of a tangled "shell" of HMF lines which always enclose the inner solar system somewhere between Jupiter and Mars.

D. THE GMF-SCR INTERACTION

The Pioneer I space probe provided the first direct measurement of the distant GMF (Sonett, et al., 1960a). It was launched on October 11, 1958, one of the quietest days on record, geomagnetically, so that it may provide a good estimate of the steady-state fields, as well as a lower bound to fluctuations. Over regions from 3.7R to 7R , and from 12.3R to 14.8R it measured the component of total GMF perpendicular to its direction of motion; however, this constituted nearly the total induction at the point. The authors write that there is "close general agreement with an extrapolated inverse-cube law within 10% [in the inner region]." In the outer region the field is about double the expected GMF, and suddenly drops off from 30 gammas to around 6 gammas at 13.6R. The relatively sudden drop in the field is unmistakable, but the resolution is so poor that one could only say that the transition region is probably less than 2000 km thick, i.e., less than 2.5% of the dimensions of the magnetosphere. The direction of the probe was close to local noon while traversing the outer region.

The Explorer X probe (IGY Bull. 1962a), measured the magnetic field as well as the SCR. At 6R an angular deviation between measured and computed fields is evident, which becomes relatively stable at a distance of 11R ; this is interpreted as indicating an appreciable extraneous field superimposed on the GMF. This effect continues out to 19R where the field no longer shows the characteristic $1/r^3$ decrease of a dipole field. The authors conclude that the GMF is negligible here; it seems more likely that distortion of the GMF close to the

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boundary by the induced field have altered the dipole character of the GMF beyond recognition. Between 20R and 21.5R the field is stable and approximately radial from the sun with a "stream angle" from the radial of 30° to 40° in the ecliptic plane. This agrees with McCracken's figure of 50° (see Fig. 2) and supports the notion that the total field at the interface must be tangential and, therefore, (at this position on the night side) nearly parallel to the SCR and the HMF embedded in it.

The actual interface appears to be at 21.5R , where the magnetic field changed dramatically from 32γ to 9γ in a distance of less than 200 km. There was also a violent change in direction. This abrupt change was coincident with the first observations of SCR. Thereafter, considerable irregular fluctuations as wide as 20γ were measured out to 25R after which they settled down to around $4 - 8\gamma$ in amplitude, superimposed on an average field of $12 - 16\gamma$. After 34.5R the field declined further to an average of around 10γ , interrupted by two regions in which the fields again rose to around 20γ , the increase being sustained for the order of an hour or more. This may be due to the fact that the trajectory is nearly parallel to the expected interface in this region (Bonett, et. al., 1962) and if the interface is slightly unstable it could cross and re-cross it several times, and apparently does re-enter the magnetosphere finally, just before apogee is reached.

The information obtained should eventually yield very useful interpretations of the interface dimensions, fields, and stability of the interface, with possible implications for a better understanding of the Van Allen radiation belts.

E. GEOMAGNETIC STORMS

Variations in the GMF are classified as S (solar), L (lunar), or D (disturbed), with numerous subclassifications, the most striking of which is the so-called D_{st}, or storm-time variation. This is the underlying world-wide systematic variation in GMF which occurs after the sudden commencement (SC) of the storm. The onset of the storm is simultaneous to within one or two minutes all over the earth, and consists mainly of

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a sudden sharp rise in the horizontal (H) component of the GMF, of the order of 20 - 30γ , which may persist for 2 to 6 hours and is known as the positive, initial, or first phase of the storm. The field then drops rapidly below its original value and the negative, or main phase begins, so-called becuase its amplitude is generally larger than that of the first phase. This phase may last for several days, slowly rising to the original value in what is known as the final, or recovery phase (Mitra, 1952). Changes in the auroral zones are generally much more severe and depend strongly upon local time.

Vertical (Z) component changes are generally much smaller than the corresponding H changes, and of opposite sign. Near the geomagnetic equator Z is unaffected by storms; however, changes in H are greatest at the geomagnetic equator.

Storms are classified as moderate (50γ), active (150γ), or great (>200γ) ; they are easily distinguished from the quiet day solar variation, S_q, which is generally of the order of $\pm 10\gamma$. The S_q variation further corroborates the concept of the GMF-SCR interaction: it is greatest during the day and in the summer hemisphere; it is 50 - 100% greater at sunspot maximum than at sunspot minimum (Mitra, 1952).

Storms have been definitely correlated with the appearance of solar flares, although there are numerous cases in which fairly strong storms have occurred without any visible solar activity. Storms are also accompanied by ionospheric disturbances and polar auroral activity.

F. RING CURRENTS

The existence of a ring current high above the earth was postulated by Chapman and Ferraro (1933) to explain the main phase of the geomagnetic storm, which could not be accounted for by SCR compressing the GMF, as in the first phase. By means of a harmonic analysis of field fluctuations in storms they showed that the main phase may be ascribed to a westward-flowing ring current circling the earth at a height of a few hundred kilometers, with intensities of the order of 100,000 amperes, depending upon the strength of the storm (Mitra, 1952). Later estimates placed the altitude of the current at around 4R to 7R .

The first direct measurements of this current were those of Explorer VI and Pioneer V rocket probes (Smith, et. al., 1960; Sonett, et.al., 1960b), which discovered anomalies in the GMF between 4R and 8R , explainable on the assumption of a ring current. With its geometry and intensity determined to give closest agreement with the data, the authors found a toroidal, westward-flowing ring current of radius 9R, circular cross-section of radius 3R, and current of the order of 5 Megamps. The axis of the ring was taken parallel to the earth's axis.

It should be noted, however, that the model was constructed on the basis of very little, and inconclusive, evidence which may only be considered strongly suggestive, at best. Later probes, Explorers X and XII, found no evidence of ring current. Nonetheless, it appears that the concept of a ring current giving rise to the main phase of the storm has much to commend it, although its existence and source remain in doubt.

G. THE AURORA AND AIR GLOW

We shall here review only those auroral phenomena which seem to be most directly related to the GMF-SCR interaction. The most important of these is the well-established correlation between auroral, magnetic, and solar activity, the eleven-year cycle being clearly evident in all three phenomena.

Auroras generally occur in high latitudes; contours of equal frequency of occurrence, or isochasms, are roughly concentric ovals, with centers approximately at the magnetic pole. The maximum-frequency isochasm in the northern hemisphere lies along 67° north magnetic latitude; a similar, but less well-known phenomenon occurs in the southern hemisphere. At this latitude the aurora averages 243 days/year, decreasing to around 200/yr at $72^\circ N$ and 100/yr at $62^\circ N$ magnetic latitude (Chapman, 1961a).

Photoelectric observations of auroral spectra during the day show that activity is largely restricted to local midnight $\pm$ 3 hours. Using the dipole as a reference axis it is found that the principal maximum of auroral intensity occurs about one hour before magnetic midnight.

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Although the auroral spectrum is made up largely of oxygen and nitrogen lines exhibiting low-temperature thermal Doppler broadening, a small part of the light comes from hydrogen α and β lines. When viewed sideways these lines show Doppler broadening, indicating thermal velocities of the order of 400 km/sec ; when viewed from below, along a GMF line which the aurora tends to follow, there is an electromagnetic Doppler shift of the entire spectrum indicating that the emitting particles are descending with mean speeds of around 500 km/sec , and some as high as 3000 km/sec (Chapman, 1961a). This is comparable to the SCR drift velocity of 300 km/sec.

We have already mentioned the correlation between auroras and geomagnetic storms; in IGY Bull. No. 45, (1961a) we find an instance of direct observation of the "dumping" of particles from the inner Van Allen belt into the auroral zones during a geomagnetic storm.

The night "airglow" or "nightglow" accounts for 50% of the luminescence of the night sky, apart from auroras. It is a sort of permanent aurora, and always present; although its intensity is small compared to auroral intensities, the question of the precise distinction between aurora and airglow is not yet resolved (Chamberlain, 1961b).

Thus far the airglow has only been observed at night; intensities are higher at twilight and before dawn than at midnight minimum by a factor of around fifteen. The "dayglow" spectrum is believed similar to that of the "twilightglow" with intensity as great or greater; however, attempted rocket observations of dayglow have resulted in contradictory estimates (Chamberlain, 1961b). In addition to this quasi-regular diurnal behavior, there is also a seasonal variation, and irregular behavior which is closely correlated with magnetic activity.

II. SOLAR CORPUSCULAR RADIATION

A. KINETIC PROPERTIES OF THE SCR

The following calculations of velocity, mean free path, and conduc‑
tivity, are based on Delcroix (1960) Chap. 11, in which it is shown that
for a strongly ionized gas an effective collision frequency is defined as

$$\nu = 0.0876 \ nT^{-3/2} \log_e \Lambda \ , \tag{1}$$

in the case of proton-proton collisions. The temperature of the ions is
T, n is their number/cm^3, and Λ is the average ratio of Debye length to
distance of closest approach for proton-proton Coulomb collisions. If
we take the Explorer X figures for the SCR at 1 a.u. (astronomical unit)
of $n = 10/cm^3$, $T \doteq 500,000 \ ^\circ K$, then

$$\log_e \Lambda = 27.9 \ ; \quad \nu = 6.90 \times 10^{-8} \ sec^{-1} \ , \tag{2}$$

and the electrical conductivity is (Spitzer, 1956, Chap. 5)

$$\sigma = 1.94 \times 10^5 \ mhos/meter \ , \tag{3}$$

considering ion-electron and electron-electron collisions in a singly-ionized
plasma. The mean thermal velocity of an ion is

$$v_i = (8kT/\pi m)^{1/2} = 103.4 \ km/sec. \ , \tag{4}$$

and the mean free path is

$$\lambda_i = v_i/\nu = 15.0 \times 10^8 \ km = 2.35 \times 10^5 \ R = 10.0 \ a.u. \tag{5}$$

Even with a more conservative estimate of $T = 10^5 \, {}^\circ K$, we find

$$\nu = 7.07 \times 10^{-7} \ sec^{-1} \ ; \quad v_i = 46.3 \ km/sec \ ,$$

$$\lambda_i = 6.54 \times 10^7 \ km = 0.44 \ a.u. \ , \qquad (6)$$

so that the mean free path of SCR ions at the earth's orbit is considerably larger than the geomagnetic cavity could possibly be, even larger than the dimensions of the inner solar system.

One may apply the same formula at the base of the solar corona, where, let us say, $T = 10^6 \, {}^\circ K$, $n = 10^8/cm^3$; then

$$\nu = 0.18 \ sec^{-1}; \ v_i = 146.4 \ km/sec; \ \lambda_i = 814 \ km; \qquad (7)$$

this last figure may be compared with the solar scale height

$$h = kT/mg_s = 31,000 \ km \ , \qquad (8)$$

where g_s is the solar gravity.

We find the thermal pressure of the SCR at 1 a.u. is

$$\text{for} \qquad T = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \times 10^5 \, {}^\circ K \ , \qquad (8.1)$$

$$p = 2nkT = \begin{bmatrix} 14 \\ 2.8 \end{bmatrix} \times 10^{-10} \ dynes/cm^2 \ , \qquad (8.2)$$

and

$$2 \ nmV^2 = 3 \times 10^{-8} \ dynes/cm^2 \ , \qquad (8.3)$$

the pressure of the SCR reflected from the front of geomagnetic cavity. Thus, on the dayside the thermal pressures are generally neglected compared to the dynamic pressure, $2nmV^2$.

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B. PARKER'S MODEL

Inspired by Chapman's (1957, 1959) hydrostatic-conductive-equilibrium model of the corona, Parker (1958a,b, 1959, 1960a,b, 1961a,b,c) has constructed a hydrodynamic continuum model of the expanding corona with which he has attempted to deduce observed coronal temperatures, densities, and expansion velocities. The best review of his work is found in Parker (1961a), although it omits many of the mathematical details.

The basic ingredients of the continuum solutions attempted by Chapman, Parker, and later Chamberlain are:

(1) neglect electromagnetic fields in the corona and consider it to be a quasi-neutral single fluid;

(2) assume spherical symmetry; neglect solar rotation;

(3) conservation of matter gives a simple integrated continuity relation in spherical symmetry,

$$nr^2 w = n_0 a^2 w_0 = \text{const.} , \qquad (9)$$

where n = number density of ions, w is the drift velocity of the expanding corona, r is the heliocentric distance from the center of the sun, and subscripts $(_0)$ refer to values at r = a, the "base" of the corona;

(4) assume an isotropic pressure tensor, although it may be expected that the high velocity of the stream and the presence of the HMF will eventually introduce anisotropies:

(5) assume an equation of state for the corona,

$$p = 2 nkT \quad \text{(perfect gas)} ;$$

(6) assume validity of the classical single-fluid Navier-Stokes equation

$$\partial \underline{w}/\partial t + (\underline{w} \cdot grad)\underline{w} = -(1/nm)grad \; p - \underline{g}_s \; ,$$

where p = pressure, m = ion mass, $\underline{g}_s$ is acceleration due to solar gravity;

(7) assume some form of the heat conduction equation to apply:

$$div \; (K \; grad \; T) = \partial Q/\partial t \; ,$$

where K is the thermal conductivity and Q is the energy density;

(8) consider only the steady-state, $\partial/\partial t = 0$.

Chapman neglected the convective term in (6), and Parker noted that this results in a pressure which approaches 10^{-4} dy/cm^2 at $r = \infty$, much larger than the interstellar pressure of 10^{-14} dy/cm^2 thought to actually exist there. Thus, the corona cannot be in static and conductive equilibrium. Parker's approach includes the convective term in (6), but avoids (7) by assuming, a priori, that for some undetermined $r = b$, there is a region $a \leq r \leq b$ over which the corona is isothermal, while its temperature decreases adiabatically for $r > b$. Thus (7) and (8) are not satisfied, and there is no conductive equilibrium. For $a = 10^{11}$ cm (1.43 R☉) Parker sets $T_0 = 10^6 \; °K$, $n_0 = 3 \times 10^7$/cm^3 .

In his analysis Parker derives a family of solutions, all of which are either physically impossible, or else yield very low drift velocities of the order of 30 km/sec , hardly the "solar wind" predicted by Biermann, but more like Chamberlain's "solar breeze." However, it is Parker's contention that there is one and only one "singular" solution which behaves properly; i.e., gives a physically meaningful solution with high velocity far from the sun and low velocity near the sun. This is the crucial issue, because Chamberlain (1960, 1961a) claims that Parker has integrated his equations incorrectly, that his "singular" solution is not mathematically valid. Chamberlain believes that a solution is either high- or low-velocity, and cannot "cross over;" that Parker's solution implies finite energy per particle at infinity, which

in turn must imply mass transport of energy so large as to make heat transfer by conduction negligible, so that the adiabatic approximation should hold almost to the corona, contradicting Parker's assumption of an isothermal corona between a and b. Furthermore, to maintain coronal temperature out to $10R_\odot$ or more requires too much energy. Despite rebuttal and counter-rebuttal the controversy over these points remains in force at the present time (1962).

Although b is at Parker's disposal, w_0 is $\underline{not}$, due to the highly "singular" nature of his solution, and the assumed value of T_0. Calculations of n, w for several combinations of T_0 and b (Parker, 1960b, pp. 856-858), yield $T_0 = 10^6$ $^\circ K$, $b = 8a = 11.4$ $R_\odot$, and $n = 20/cm^3$, $w = 310$ km/sec at 1 a.u., which is in very close agreement with the Explorer X space probe results (see I.D.) and was published six months before the launching of that probe. In addition, it should be noted that for $T_0 = 1.22 \times 10^6$ $^\circ K$, Parker's model of the isothermal corona exhibits agreement with density observations based on electron scattering which is, in his words, "...so extraordinary as certainly to be fortuitous." But it is very suggestive, nonetheless.

However, there are numerous difficulties, including the fact that by using Parker's figures, $T_0 = 1.22 \times 10^6$ $^\circ K$, and $n_b = 2 \times 10^4/cm^3$, his adiabatic temperature relation,

$$(T/T_0) = (n/n_b)^{2/3} ; \qquad (10)$$

yields $T = 16,000$ $^\circ K$ for $n = 30/cm^3$ at 1 a.u. A figure which may be compared with Chamberlain's (15-20,000 $^\circ K$), Chapman's (440,000 $^\circ K$) and the Explorer X results ($10^5 - 10^6$ $^\circ K$). The similarity to Chamberlain, who also uses a hydrodynamic approach, and the disagreement with empirical estimates coupled with the electron-density agreement near the sun, and with the values of ion mean free path calculated in the previous section (II.A), indicate that the classical continuum approach may indeed be valid near the sun, but that the large mean free paths at 1 a.u. require that we go over to a free-molecule flow at some distance between the sun and the earth.

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Parker derived an expression for the HMF by assuming that particles leave the sun with constant velocity and drag the HMF lines with them. Thus, in the system rotating with the sun these particles describe spiral arcs (the "garden-hose" effect) and the HMF must be tangent to these arcs. By combining this requirement with div $B = 0$, he finds field lines in the plane of the ecliptic being stretched out and bent by the SCR, with radial component depending on $1/r^2$, the azimuthal component on $1/r$. Parker, (1959) states: "The field at the orbit of the earth is very nearly radial.... The field is unstable because of the anisotropic expansion of the gas as it comes out from the sun. The collision rate is extremely low; hence if the gas expands in directions perpendicular to the radius, but not in the radial direction, its thermal motions become anisotropic and the gas itself becomes unstable. The field becomes disordered." This, in Parker's view, forms the magnetic shell which he feels is responsible for the diffusive decay of cosmic rays. The Forbush decrease in galactic cosmic rays during a geomagnetic storm is explained by the fact that storm particles are more energetic, their stream lines have less curvature and stretch out the HMF forming a magnetic shock wave (McCracken, 1962).

C. CHAMBERLAIN'S MODEL

Initially, Chamberlain, (1960), applied Jeans' evaporative theory to the calculation of the expanding solar corona, and more recently (Chamberlain, 1961a), he has adopted Parker's approach, but with a fundamental difference: Chamberlain has used a heat equation derived from the first law of thermodynamics

$$dq = du + pdv , \qquad (11)$$

where q, u, v are heat absorbed, internal energy, and volume, respectively, per unit mass. Since the heat absorbed per unit volume per second is given by Fourier's heat-conduction equation to be $\text{div} (K \text{ grad } T)$, then the heat absorbed per gram-second is (Liepmann and Roshko, 1957, Chap 13):

$$(1/mn) \; \text{div} \; (K \, \text{grad} \; T) \;\; = \;\; dq/dt \;\; = \;\; du/dt + p \; dv/dt \;\; . \qquad (12)$$

If the entire system is moving we may replace the d/dt operator with the mobile operator $\partial/\partial t + \underline{w} \cdot \text{grad}$ measured by the stationary observer. Since $u = 3kT/2m$ and $v = 1/mn$, we set $\partial/\partial t = 0$ and find:

$$\text{div} \; (K \; \text{grad} \; T) \;\; = \;\; (3nk/2) \; \underline{w} \cdot \text{grad} \; T - kT\underline{w} \cdot \text{grad} \; n \;\; , \qquad (13)$$

where the gas is understood to be monatomic. Thus, if he takes $K = K_0 T^{5/2}$, Chamberlain finds representative values of $\leq 21{,}500 \; {}^{\circ}K$ and $n = 30/cc$ at one a.u. These are comparable to the values obtained by the evaporative model, but far from the empirical estimate of $10^5 - 10^6 \; {}^{\circ}K$. However, Chamberlain has initially assumed the vanishing of particle expansion energies at ∞, so that his "breeze" conclusion is to be expected, and, of itself, does not preclude a solar wind, only its inevitability as the result of Parker's "singular" solution. This paper is to be recommended for its clarity.

Although the ion mean free path at 1 a.u. is of the order of an astronomical unit, it may not necessarily invalidate a continuum approach, since the Larmore radius of the protons in the SCR is 3000 km at 1 a.u. for a HMF of 1γ , a relatively small length. Thus particles revolve about field lines which transmit the stresses of the coronal expansion throughout the inner solar system. Thus, one may perhaps treat the problem near 1 a.u. , by means of an anisotropic pressure tensor, as suggested by Bonetti, et. al., (1962.)

III. GEOMAGNETIC STORMS

A. FIRST PHASE

1. Early Work

At first glance it may appear out of order to discuss the nonsteady
GMF-SCR interaction before investigating the steady-state; but, histori-
cally, it was the nonsteady phenomena which first attracted the attention
of science, and eventually led, within the last decade, to the concept of
the steady-state interaction. In 1892, Lord Kelvin demonstrated that the
storms could not be directly due to variations in the HMF, and Hale's
measurements confirmed this fact so that the interaction had to be either
corpuscular or radiative (ultra-violet). In 1896, Birkeland showed that
electron beams shot out of the sun would be deflected by the GMF, by pro-
jecting cathode rays toward a magnetized sphere with the exotic name of
Terrella. Impact was generally in the polar regions, (Mitra, 1952, p. 464),
and since then the theories of geomagnetic storms and auroral phenomena
have been linked together.

Following this, Störmer developed the theory of orbits of individual
charged particles in a magnetic dipole field. By 1931, Chapman and Ferraro
(1931a, p. 80) were able to state that "...our work confirmed Lindemann's
conclusion that the only admissible kind of stream is one that is electro-
statically neutral to a very high degree of approximation." Mitra (1952,
Chap. IX) offers an ample review of the work of Birkeland and Störmer;
however, the answers to the various problems discussed in this review are
not to be found in the realm of classical single-particle physics, although
such methods have afforded us numerous valuable insights.

Most of the theoretical foundation for present thought on the sub-
ject of the GMF-SCR interaction is based on the work of Chapman and Ferraro,
collectively and individually, in which they attempted to explain the origin
and behavior of magnetic storms by means of certain mathematical idealiza-
tions. The first such was the application of Maxwell's solution (Maxwell,
1881), for a magnetic dipole moving parallel to a perfectly conducting

plane sheet. This showed the lines of force to be parallel to the boundary except at two neutral points, N, where the field vanishes (see Fig. 3). The image dipole doubles the field component parallel to the boundary and concels the normal component, but it vanishes within the conductor.

Chapman and Ferraro tried to demonstrate the existence of the geomagnetic cavity and ascertain its size along the earth-sun line by considering the forces on a "surface layer" due to accumulation of matter, change of momentum of matter entering the layer, and magnetic stresses from the augmented GMF inside the cavity. The results obtained then, by laborious approximation, agree very closely with more recent results obtained by Ferraro (1952, Section 5.6) with the aid of an electronic computer. However, both treatments of this difficult, time-dependent problem fail to account satisfactorily for continuity relations at the surface (see IV.A.) and give rise to a very fundamental contradiction concerning momentum balance across the surface layer. However, their concept of the geomagnetic cavity has been borne out by recent measurements.

As a result of their investigations Chapman and Ferraro developed a model of the GMF-SCR interaction which also embodied the germs of the ring current idea, and a possible explanation of the correlation with auroral activity. They concluded, (1931a, p.83): "In the case of a stream from the Sun advancing towards the Earth, a hollow space round the Earth is formed in the stream...open at the back of the Earth...part of the stream which has collected near the Earth remains for a time, probably in the form of a ring around the equator; this gradually disappears by the passage of the ions and electrons along the Earth's lines of force, into the atmosphere in high latitudes."

2. The Cylindrical Model

In 1940 Chapman and Ferraro presented the cylinder model, in which a cylindrical sheet of plasma collapses from infinity upon a coaxial magnetic field (see Fig. 4). The original (permanent) field is
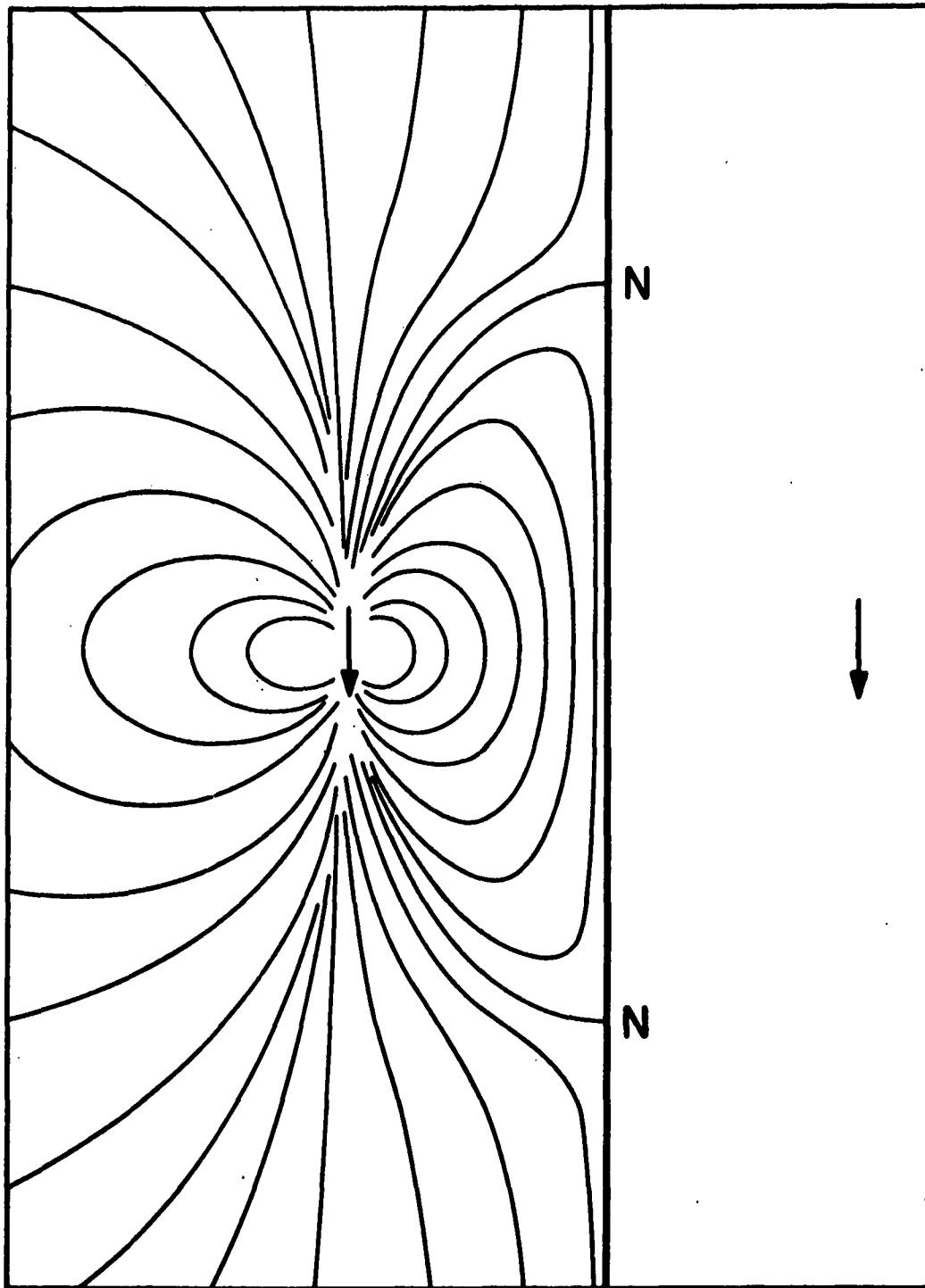
FIG. 3--Image Dipole Solution.
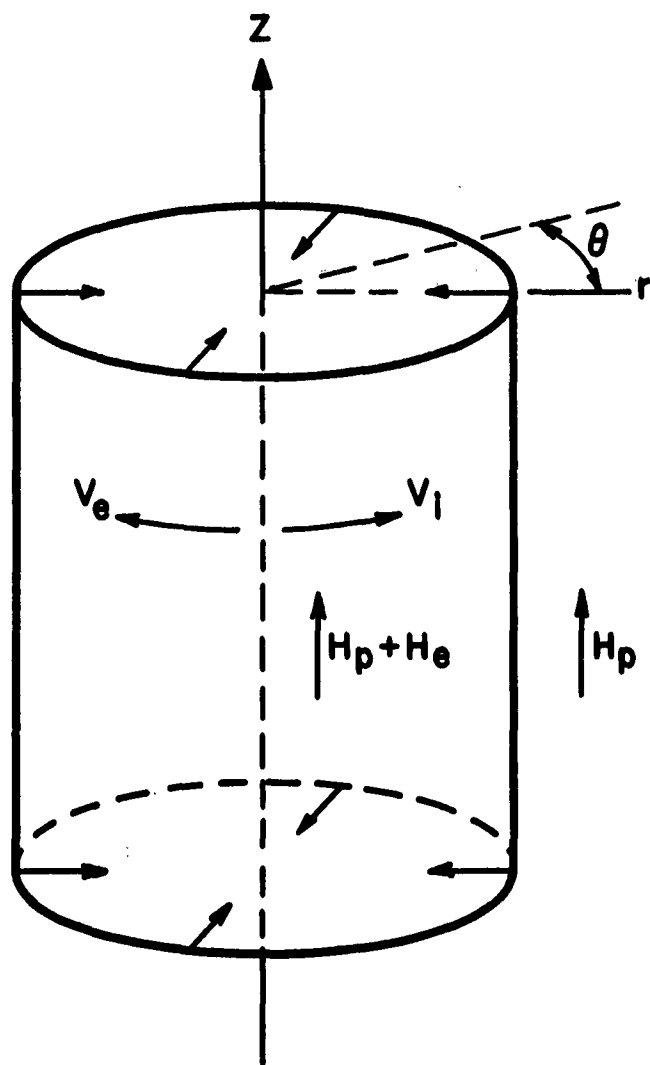
FIG. 4--The Cylinder Model.

$$(n > 2) \quad H_p = H_0(R/r)^n; \quad A_p = (- H_0R/(n - 2)) (R/r)^{n-1} \quad , \quad (14)$$

where H_p is in the z-direction, and its vector potential, A_p, is in the ϕ-direction, tangential to the cylinder. The sources of H_p are taken to be completely unaffected by the passage of the current sheet.

In addition to Maxwell's equations, the authors employ the equations of motion for the individual particles subject to the Lorentz force. The fields are derived from an electric scalar potential $\phi = \phi(r)$ and a magnetic vector potential $A = A(r)$ in the θ-direction. The ϕ is to account for electrostatic forces due to possible charge separation; the vector potential A includes the effect of ion and electron currents in the cylinder.

Although the authors' calculations are straightforward, they are involved; a simpler calculation which makes use of their insights suffices to reproduce the main features of their analysis. The electromagnetic Hamiltonian of a particle of charge q, mass m, is:

$$\mathcal{H} = (1/2m) (\underline{p} - q\underline{A}/c)^2 + q\phi = (m/2) (u^2 + v^2) + q\phi = mU^2/2 \quad , \quad (15)$$

where the particle velocity in (r, θ, z) - coordinates is $\underline{v} = (u, v, 0)$, and at infinity, $\underline{v} = (U, 0, 0)$. Since $\phi = \phi(r)$, the θ-component of the canonical momentum is a constant, zero in this case because A and v vanish faster than $1/r$ at infinity; therefore,

$$mv = - qA/c \quad , \tag{16}$$

and thus,

$$m_e v_e = - m_i v_i \quad \begin{pmatrix} e = \text{electrons} \\ i = \text{ions} \end{pmatrix} ; \tag{17}$$

i.e., the angular momentum of the cylinder is conserved, which might have been anticipated on grounds of symmetry.

On the basis of Eq. (17), one may assume the current J to be due to the electrons,

$$J = J_e = -Qv/2\pi cr \quad ; \quad A_e = -Qv_e/c \quad , \tag{18}$$

where Q represents the amount of positive charge per unit length, and A_e is the magnetic vector potential due to the electrons evaluated just inside the surface of the cylinder.

From Eq. (16), we have

$$m_e v_e = (e/c) (A_p + A_e) \quad . \tag{19}$$

Substituting for A_e from Eq. (18) yields

$$m_e v_e = -[cm_e R^n H_0/(n-2) aQr^{n-1}] \quad , \tag{20}$$

where $a = 1 + m_e c^2/eQ$; Chapman (1960) shows that $a \doteq 1$ for all cases of geomagnetic interest. The induced field inside the cylinder is

$$H_e \doteq 4\pi J_e = [2H_p/a(n-2)] \quad . \tag{21}$$

One may now consider the inward radial motion of the cylinder as a whole, the forces upon it given by the difference in magnetic pressure

across the surface. By symmetry, $u = u(t) = u[r(t)]$, so that the equation of motion per unit length of cylinder is $(a = 1 , n = 3)$:

$$(Qm_1/e) \, du/dt = (Qm_1/e)u \, du/dr = 2\pi r(H_e^2 + 2H_e H_p)/8\pi \quad . \tag{22}$$

Since $H_e = 2H_p$, integration yields

$$U^2 - u^2 = (eH_0^2 R^2/Qm_1) (R/r)^4 \quad , \tag{23}$$

and setting $u = 0$, we find the stopping radius, r_0 :

$$(r_0/R)^2 = (e/Qm_1)^{1/2} (H_0 R/U) \quad . \tag{24}$$

The total increase in magnetic energy per unit length at the stopping radius, $R = r_0$, is

$$\delta E_{mag} = (\pi r_0^2/8\pi) [(H_{p0} + H_{e0})^2 - H_{p0}^2] = (Qm_1 U^2/e) \quad , \tag{25}$$

or twice the available kinetic energy. Chapman (1960, p. 924) has noted this "apparent paradox" which he says is due to postulating that the permanent field is unaffected by the passage of the cylinder, so their mutual inductance is zero, and one considers only the induced field; i.e.,

$$\delta E_{mag} = (r_0^2/8) H_{e0}^2 = (Qm_1 U^2/2e) \quad .$$

However, it is probably more correct to say that the additional energy is supplied by the sources of the field in order to maintain the permanent field during the passage of the cylinder.

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It might be possible to test this theory in its essentials by covering a thin metal cylinder with a very uniform shaped charge of explosive, and placing a uniform magnetic field inside by means of a large magnet and a search coil at its center to measure $\partial B/\partial t$. The explosion should vaporize the metal and drive it inward, giving rise to a situation much like that treated by Chapman and Ferraro.

As an approach to the real physical situation, the shortcomings of this model are evident; however, it constituted an important step forward in establishing the current view that the SCR is retarded and re-flected from the GMF as well as providing some theoretical background for Ferraro's 1952 paper. It also provided the clear indication that the size of the geomagnetic cavity would be determined by a pressure balance across its boundary, involving the magnetic field on one side and the change in particle momentum on the other. The concept of "specular" reflection of particles arises simply as a consequence of the symmetry: since the potentials involve only one coordinate, r, the trajectories of individual particles must be symmetrical with respect to their perigee radius, giving the effect, at large distances, of specular reflection.

More recently, Chapman and Kendall (1961b) have made a calculation of the motion of the collapsing cylinder in the case that the cylinder particles are projected obliquely inward, rather than radially. The purpose was to obtain some idea of the shape of the GMF-SCR interface in the equatorial plane.

3. **The Plane Model**

In 1952, Ferraro published a plane model of a geomagnetic storm in which an infinite stream of plasma at $T = 0^{\circ}K$ advances in the $(-z)$ direction into a permanent magnetic field $H_p = H_0(R/z)^3$ directed in the y-direction (see Fig. 5). By assuming a total vector potential $A = A(z)$ in the x-direction, and an electrostatic field derivable from a scalar potential $\phi = \phi(z,t)$ due to charge separation, Ferraro derives approximate solutions from the time-dependent single-particle Lagrangians in a manner similar, in many respects, to the solution of the Cylindrical Model. However, since the stream is continuous, and
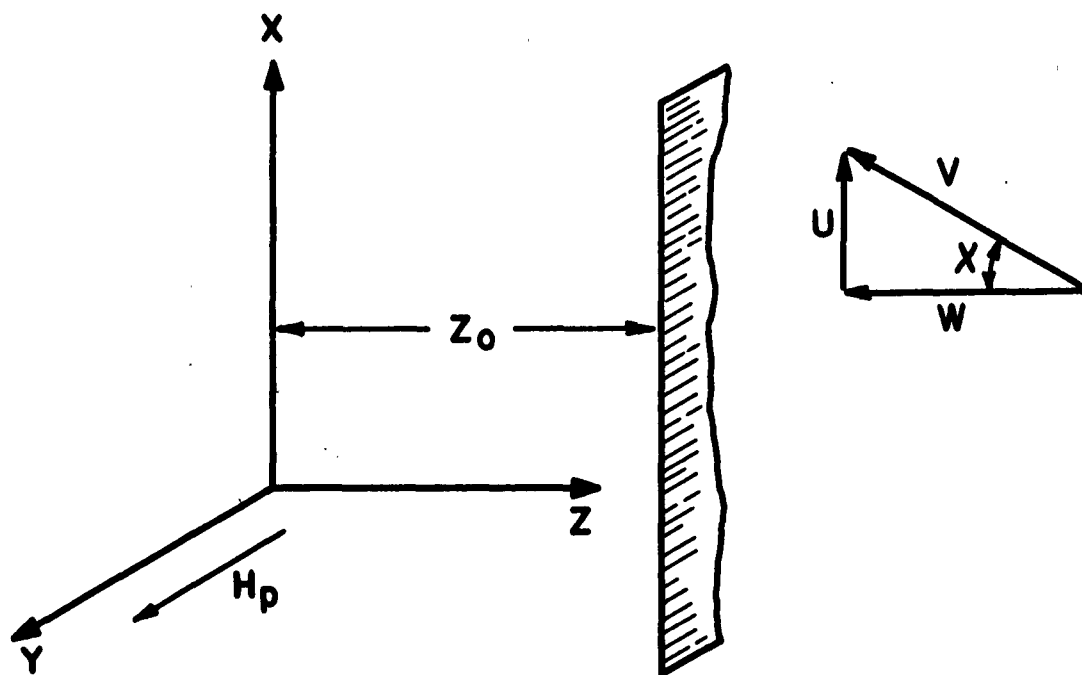
FIG. 5--The Plane Model.

infinite, and the stream front is in motion, the time-dependence greatly complicates matters.

Ferraro derives the equation

$$\partial^2 A/\partial z^2 = 4\pi(e/c)^2 (n_e/m_e + n_i/m_i)A \ , \qquad (26)$$

and after assuming $n_e \doteq n_i \doteq n$ he derives an approximate representation for A by the WKB method:

$$A = \alpha \lambda^{-1/2} \exp[- \int_{z_0}^{z} \lambda \, dz] \ ; \ \lambda^2 = 4\pi n e^2/m_e c^2 \ . \qquad (27)$$

By proceeding from this, he is then able to demonstrate that $\lambda \doteq$ const., with a possible change of the order of unity right at the surface. However this is, in effect, implied by the WKB representation which requires that $\partial\lambda/\partial z$ be very small. In fact, if particles are to be reflected from the surface of the stream (Ferraro showed that this is to be expected), then the density must become infinite at the surface in order to satisfy the continuity relations. This will be discussed in greater detail in IV.A.

The net effect of this implied assumption of constant density is that Ferraro obtains the boundary condition at $z = z_0$:

$$H^2/8\pi = n m_i W^2 \ , \qquad (28)$$

which is not a true momentum balance. Ferraro explains the discrepancy thus (p. 36): "Since the motion of the particles is reversed...the change of momentum is $2nm_i W^2$.... The result implies that one-half of the momentum of the stream is taken up by the magnetic field, and the other half by the electrostatic field." Since the electrostatic field is internal to the plasma, this is not a satisfactory explanation. Dungey has shown that the proper relation is indeed

$$H^2/8\pi = 2nm_i W^2 \quad , \tag{29}$$

and the difference is due to the implied assumption of constant density.

Ferraro also inferred that for an oblique flow

$$H^2/8\pi = n\, m_i W^2 \cos^2 \chi \quad , \tag{30}$$

when the flow is inclined at an angle χ to the z-axis. (Note: $\cos \chi$ is not squared in Ferraro's Eq. (159); this is a typographical error, and it is corrected in Ferraro, 1960b).

It should be noted that, for constant λ,

$$A = A_0 \exp\left[-\lambda(z - z_0)\right] \quad , \tag{31}$$

a typical electromagnetic shielding formula, so that $1/\lambda$ gives a lower (no dissipation) estimate for the "skin depth" or thickness of the interface between the GMF and the SCR. This is in agreement with the later work of Dungey on the steady-state case.

B. MAIN PHASE OF STORM

1. Ring Currents

The work described so far has served only to establish theoretical foundations to explain the augmentation of the GMF during the first phase of a geomagnetic storm. However, this does not explain the observed decrease in the GMF (main phase) which follows within a few hours of sudden commencement (SC). To account for this Chapman and Ferraro (1933) postulated a westward-flowing equatorial ring current at several earth radii due to charged particles penetrating the GMF from the approaching SCR. The effect is to produce a magnetic field opposed to the GMF; however, among other things, the theory does not

explain the time lag or the requisite stability of the circular ring in an assumed magnetic dipole field. Nevertheless, it did provide a reasonable model for the main phase decrease.

In 1957, Singer, in a brief, but comprehensive review of magnetic storm theories, described the theory of the interplanetary shock wave due to gas erupting from the sun. The impact of this shock wave upon the GMF compresses the field and also causes a magnetohydrodynamic shock within the magnetosphere which registers at the earth's surface at the auroral zones, being "channeled" there by the magnetic field lines. As the lines of force converge toward the poles the shock wave converges with them, increasing in strength about five times to a point where it can produce the auroral effects generally associated with SC. Several hours later, the gas from the solar flare which acts as the "driver" for the shock arrives at the GMF, penetrates into the field and is trapped in the magnetic tubes of force. What follows is a simple application of the theory of charged-particle drifts in anisotropic magnetic fields, where the anisotropy may be introduced by means of an electric or gravitational force, or through the inhomogeneity and curvature of the magnetic field itself.

Singer (1957), calculates the drift of protons of velocity 2000 km/sec injected perpendicular to the field lines, based on the inhomogeneity of the dipole field, neglecting magnetic effects due to the particle drifts themselves. By combining this motion with oscillation along field lines he finds a westward-flowing ring current of roughly diamond-shaped cross-section, with maximum density in the equatorial plane and around 8R, extending 30° north and south of the equator. Despite the roughness of the calculations they clearly show that particle drifts can give rise to magnetic fields sufficient to produce the main phase of the storm.

There still remains doubt as to the cause of the drifts: how can the SCR penetrate the GMF if the latter has already been compressed due to the prior arrival of the shock wave? Singer only says that particles may leak into the field if it is sufficiently distorted from the magnetic dipole, presumably in higher latitudes where the GMF disturbance depends more strongly on local time. Dessler, et. al., 1961, removed this objection

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maintaining that penetration is not necessary: instead, large-amplitude hydromagnetic waves are generated by the impact of the SCR upon the GMF; the resulting impulse is transmitted in the form of longitudinal hydromagnetic shock waves traveling toward the earth, and transverse waves traveling toward the poles along magnetic field lines.

Like ordinary waves in a compressible fluid (Liepmann and Roshko, 1957, Chap. 3), these shock waves tend to steepen as they propagate, until the extreme temperature and density gradients across the shock dissipate energy fast enough to prevent further steepening. Thus, as the shock passes through the gas, the gas is heated, and the higher thermal energies of the gas particles then give rise to higher drift velocities in the GMF, due to field inhomogeneities and curvature of field lines. The latter also gives rise to a westward-flowing ring current. The heating goes on until the gas pressure is comparable to that of the magnetic field perturbation, which in turn is equal to the change of momentum of the incident SCR. Thus the gas particles acquire thermal velocities of the order of the velocity of the SCR; the resultant situation is similar to that which Singer obtains by means of injection.

When the ratio, $\delta B/B$, of perturbed field to steady field becomes much less than unity, the hydromagnetic heating decreases rapidly, so that the authors estimate the maximum ring current density to occur around $4R$. The calculations which substantiate the theory are only order-of-magnitude, and the theory offers ample scope for further research.

Although the existence of a ring current as the source of the main phase decrease is still in doubt, Slutz (1962), has noted that if it does exist the pressure of its magnetic field would enlarge the size of the geomagnetic cavity, since it would augment the GMF outside the ring. Thus, we find the minimum dimension of the cavity (subsolar point, equatorial plane) from the boundary condition [Eq. (29)], where H equals the dipole field plus the induced field due to surface currents, which we shall take equal to the original field

$$4H_0^2 \, (R/r_0)^6 \;=\; (8\pi) \; 2nm_1 W^2 \;\; , \tag{31}$$

where $H_0 = 0.3$ gauss. This yields

$$r_0 = 8.85R \quad , \tag{32}$$

which is smaller than the experimental estimates which are in the neighborhood of $13.6R$. However, if, on the basis of the rather fragmentary Explorer VI and Pioneer V results, we assume a toroidal ring current in the equatorial plane and coaxial with the GMF, with radius $9R$ and cross-section radius, $3R$, and current of 5×10^6 amps. , then Eq. (31) is altered:

$$4[H_0(R/r_0)^3 + H_{ring}]^2 = (8\pi) \, 2nm_i W^2 \quad , \tag{33}$$

again assuming the effect of the surface currents is to double the original field at the boundary. If we calculate H_{ring} from the formula for a current loop we find that $r_0 = 12.2 \, R$ (Smythe, 1950, p. 271).

2. Aurorae

We have seen in the preceding section how Singer and Dessler could explain auroral activity in terms of the propagation of hydromagnetic shocks along field lines into the auroral regions. One need not have a geomagnetic storm to accomplish this; it would be sufficient for the interface to be easily distorted under the influence of random fluctuations in the steady-state SCR. Thus, fluctuations above some minimum threshold energy might generate the hydromagnetic waves which give rise to the "steady-state" aurora which occurs most of the time.

It has also been argued that there are "neutral points" in space corresponding to the points "N" in Fig. 3, at which SCR particles can enter the magnetosphere and spiral down the field lines toward the poles, where they give rise to the aurora by collisions with atmospheric constituents. This does not explain the fact that auroral activity is a maximum on the nightside, unless the fields are sufficiently twisted near the poles, due to the rotation of the earth.

Chapman, (1961a), believes that high-energy particles in the SCR, especially during storms, may break through the GMF-SCR interface and become trapped, replenishing and distorting the Van Allen belts. Then, at those points on the night side of the magnetosphere where the ring current field cancels the GMF, one finds internal neutral points where particles from the radiation belts may spiral down to the earth. These neutral points form long luneshaped strips, which explains why the aurora shows a characteristic ribbon-like structure. The dimensions of these strips may be 100 x 1000 x 1 miles . On the day side there are no neutral points or strips since the GMF is compressed by the SCR.

There is as yet no generally accepted theory; the question remains open.

IV. STEADY-STATE GMF-SCR INTERACTION

A. ONE-DIMENSIONAL MODEL

Dungey (1958, Chap. 8) applied Ferraro's plane model to the calculation of the steady-state interaction, in which $\partial/\partial t = 0$ and the plasma-field interface is stationary in time, i.e., $dz_0/dt = 0$. We shall here expand Dungey's analysis in detail, since it constitutes our best mathematical justification for some basic ideas about the interaction between the GMF and the SCR. In addition, we present calculations of particle trajectories and currents.

Following Ferraro's model, consider a semi-infinite, neutral, fully-ionized stream of protons and electrons moving in the negative z-direction as shown (Fig. 5). The particles are taken to be at absolute zero temperature, and penetrate into the permanent magnetic field H_p as far as the plane surface of the plasma at $z = z_0$.

We assume that H_p points in the y-direction, and is given by

$$H_p = H_0(a/z)^3 , \tag{34}$$

where $H_0 = 0.3$ gauss which is augmented by the magnetic field H' in the y-direction, due to surface currents in the $z = z_0$ plane.

We assume the existence of stationary electric and magnetic fields in the plasma, which may be represented by the potentials

$$\phi = \phi(z) \; ; \; A = A(z) , \tag{35}$$

where A is the vector potential, and is chosen to be pointing in the x-direction, so that the magnetic field $H = \text{curl } A$ within the plasma will be parallel to $H_p + H'$, and boundary conditions can be satisfied.

The particles will be moving at $z = \infty$ with only a z-component of velocity; there are no forces ($E = \partial\phi/\partial z$ is in z-direction) which can produce a y-component, so that the velocity of a particle is represented by

$$v = (u, 0, w) \ . \tag{36}$$

If q is the charge on a particle in the plasma and m its mass, its Hamiltonian is

$$\mathcal{H} = (2m)^{-1} (p - \frac{qA}{c})^2 + q\phi = \tfrac{1}{2}m (u^2 + w^2) + q\phi \ , \tag{37}$$

and since $\mathcal{H}$ does not depend explicitly on time its value is a constant; thus

$$\mathcal{H} = (m/2) (U^2 + W^2) \ , \tag{38}$$

where W is the particle velocity in the negative z-direction and U is the velocity in the x-direction at $z = \infty$, the same for electrons and protons.

Since the densities of interest are so low, one is justified in neglecting inter-particle collisions; by also neglecting any radiation through bremstrahlung we have constructed a conservative Hamiltonian in which $(u^2 + w^2)$ depends only on z. The property of time-reversibility shows that the particle trajectories will be symmetric with respect to a line parallel to the z-axis drawn through their turning-points in the z_0 plane. Thus the absolute value of $w(z)$ for both incoming and outgoing particles of a single type is the same, although different in sign; however, $u(z)$ does not change sign.

The charge density at some value of z is due to the density of both incoming and outgoing particles. For charge density ρ and current density J:

$$\rho = 2e(n_i - n_e) \ ; \ J_x = \frac{2e}{c} (n_i u_i - n_e u_e) \ ; \ J_y = 0 = J_z \ . \tag{39}$$

Maxwell's equations take the form

$$\text{div } E = 4\pi\rho = 8\pi e(n_i - n_e) \ , \tag{40}$$

and

$$\text{curl } H = 4\pi J_x = 8\pi \frac{e}{c} (n_i u_i - n_e u_e) \ . \tag{41}$$

In the geomagnetic region, $z < z_0$, the existence of a permanent magnetic field can be thought of as due to a distribution of permanent magnetization, or current dipoles, which has the vector potential (in the x-direction):

$$A_p = -\tfrac{1}{2} H_0 \, a(a/z)^2 \ . \tag{42}$$

The field is augmented by H' due to the J_z current in the plasma. Since there are not sources of H' in the geomagnetic region, and since symmetry requires that its vector potential be in the direction of x and dependent upon z, the magnetostatic equation

$$\frac{\partial^2 A'}{\partial z^2} = 0 \ , \tag{43}$$

yields

$$A' = H'z \ , \tag{44}$$
$$H' = \text{const.} \ .$$

Finally, when the solution is derived the following boundary conditions must be satisfied:

(i) vector potential continuous across the boundary
   $z = z_0$;
(ii) magnetic field intensity continuous across the boundary;

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(iii) electrostatic potential continuous across the
       boundary;

 (iv) conservation of energy;

  (v) over-all charge neutrality;

 (vi) no fields at $z = \infty$, where ion and electron
       densities are equal to n_0, and velocity equals
       W in the negative z-direction, U in the x-direction .

The unknown quantities are the velocities, potentials, and densities;
they are first derived as functions of the potential, A, beginning with
the lateral velocities. Hamilton's equations of motion yield

$$\frac{-dp_x}{dt} = \frac{\partial \mathcal{H}}{\partial x} = 0 \quad ,$$

whence

$$p_x = mu + qA = const. \quad , \tag{45}$$

and condition (vi) requires that

$$U - u = + \frac{qA}{mc} \quad , \tag{46}$$

which is position-dependent, as assumed in Eq. (35).

The velocity components in the z-direction can be found from the
original Hamiltonian, Eq. (37):

$$(m_i/2)w_i^2 = (m_i/2)W^2 + eAU/c - e^2A^2/2m_ic^2 - e\phi \tag{47}$$

$$(m_e/2)w_e^2 = (m_e/2)W^2 - eAU/c - e^2A^2/2m_ec^2 + e\phi \quad , \tag{48}$$

where we have substituted for u from Eq. (46). We now make the assumption

of "quasi-neutrality;" i.e., an electrostatic field due to charge separation does exist, represented by $-$ grad ϕ, but a sufficiently strong field may be obtained from relatively little change in density, i.e.,

$$n_i = n_e = n \quad .$$

<div align="right">(49)</div>

In the steady state condition the divergence of the electric current density must be zero in both incoming and outgoing streams, so that Eq. (49) and condition (vi) lead to

$$w_i = w_e = w \quad .$$

<div align="right">(50)</div>

Since Eq. (49) would appear to violate Eq. (40), the criterion for validity of this approximation is

$$\frac{(n_i - n_e)}{n} = \frac{\text{div } E}{8\pi n e} \ll 1 \quad .$$

<div align="right">(51)</div>

The results of the calculation are consistent with this criterion.

If we substitute Eq. (50) into Eqs. (47) and (48), we find:

$$\phi = \left(\frac{e}{2}\right) \frac{A^2}{c^2} \left(\frac{1}{m_e} - \frac{1}{m_i}\right) + AU/c \quad ,$$

<div align="right">(52)</div>

and

$$\frac{w^2}{W^2} = 1 - \left(\frac{A}{A_0}\right)^2 \quad ,$$

<div align="right">(53)</div>

where $A_0^2 = m_i m_e W^2 c^2/e^2$ is just the value of A^2 when $z = z_0$, derived by setting $w = 0$ in Eqs. (47) or (48). For simplicity, we shall set

$$\alpha = \frac{A}{A_0} = \alpha(z) \quad ; \quad A_0 < 0 \quad ;$$

α will prove to be the most convenient variable for this analysis.

The particle density is found from the equation of continuity which must hold for each stream separately, so that

$$nw = n_0 W \quad ; \tag{54}$$

therefore,

$$\frac{n}{n_0} = (1 - \alpha^2)^{-\frac{1}{2}} \quad . \tag{55}$$

It is now possible to calculate $\alpha(z)$ by substituting Eqs. (46) and (55) into Eq. (41). The result is

$$\left(\frac{1}{A_0}\right) \text{curl } H = -\frac{d^2\alpha}{dz^2} = -\left(\frac{8\pi e^2 n_0}{Mc^2}\right) \alpha(1 - \alpha^2)^{-\frac{1}{2}} = \frac{4\pi J_x}{A_0} \quad , \tag{56}$$

where M is the reduced mass, $M = m_i m_e/(m_i + m_e)$. The first integration of this equation subject to condition (vi) yields

$$\left(\frac{1}{2}\right)\left(\frac{d\alpha}{dz}\right)^2 = \left(\frac{8\pi e^2 n_0}{Mc^2}\right)[1 - (1 - \alpha^2)^{\frac{1}{2}}] \quad . \tag{57}$$

Since $H = A_0(d\alpha/dz)$, the condition which determines the boundary ($\alpha = 1$) is that incoming particles are specularly reflected by the pressure of the __total__ magnetic field (see Fig. 5):

$$\frac{H^2}{8\pi} = 2n_0(m_i + m_e) W^2 = 2n_0(m_i + m_e) V^2 \cos^2 \chi \quad . \tag{58}$$

It should be noted that Ferraro finds $H^2/8\pi = 1 \cdot n_0(m_i + m_i)W^2$, which violates conservation of momentum. This is because Ferraro in essence considers n = constant, and does not account for its sudden increase at the boundary; this assumption leads to

$$\lambda^2 = 8\pi n_0 e^2/Mc^2 \quad ; \quad d^2\alpha/dz^2 = -\lambda^2\alpha \quad ; \quad \alpha = e^{-\lambda(z - z_0)} \quad , \tag{59}$$

where $1/\lambda$ is the ion Larmor radius at $z = z_0$.

The fields in the region $z < z_0$ may now be determined by conditions (i) and (ii), which yield

$$H' - \left(\frac{H_0}{2}\right)\left(\frac{a}{z_0}\right)^3 = \frac{A_0}{z_0} \tag{60}$$

$$H' + H_0 \left(\frac{a}{z_0}\right)^3 = 4[\pi n_0(m_i + m_e)W^2]^{\frac{1}{2}} \quad . \tag{61}$$

Since $z_0 > a$, a simple numerical calculation shows that the right side of Eq. (60) is negligible compared with the magnitudes of the terms on the left, resulting in

$$H' = \left(\frac{H_0}{2}\right)\left(\frac{a}{z_0}\right)^3 \tag{62}$$

and

$$\left(\frac{9}{4}\right) H_0^2 \left(\frac{a}{z_0}\right)^6 = 16\pi n_0(m_i + m_e)W^2 \quad . \tag{63}$$

To find the vector potential in the region $z > z_0$, Eq. (57) is integrated:

$$(e^{\sqrt{2}}) \exp[-\lambda(z - z_0)] = \left[\frac{\tan(\theta/4)}{\tan(\pi/8)}\right] \exp[2\cos(\theta/2)] \quad, \quad (64)$$

where $\sin\theta = \alpha$. Figure 6 compares Ferraro's exponential solution with Dungey's exact solution.

Explorer X (IGY Bull., 1962a, p. 13) observed a flux of SCR of $n \doteq 15/\text{cm}^3$ and $W \doteq 700$ km/sec which coincided with the sudden commencement of amplitude 300γ observed at College, Alaska. Under these conditions Eqs. (62) and (63) yield $H' \doteq 80\gamma$ when the front of the storm stream comes to rest in the GMF. This constitutes a fair approximation, considering that the actual field is curved, not planar, and would be magnified in the auroral zones according to Singer (see III. B.1).

If we consider the trajectory of a single incoming particle of charge q, mass m, with time coordinate $-\infty < t < \infty$ in a system moving with $v = (U, 0, 0)$, then the arc in space is described by

$$\frac{dx}{dz} = \left(\frac{dx/dt}{dx/dt}\right) = -\left(\frac{qA_0}{mWc}\right) \alpha(1 - \alpha^2)^{-\frac{1}{2}} \quad . \quad (65)$$

Equation (56) allows us to replace the right side of Eq. (65) by

$$\frac{dx}{dz} = \left(\frac{qA_0}{m\lambda^2 Wc}\right) \frac{d^2\alpha}{dz^2} \quad (66)$$

$$x = \left(\frac{qA_0}{m\lambda^2 Wc}\right) \left(\frac{d\alpha}{dz}\right) \quad , \quad (67)$$

and substituting into (67) from Eq. (57) yields
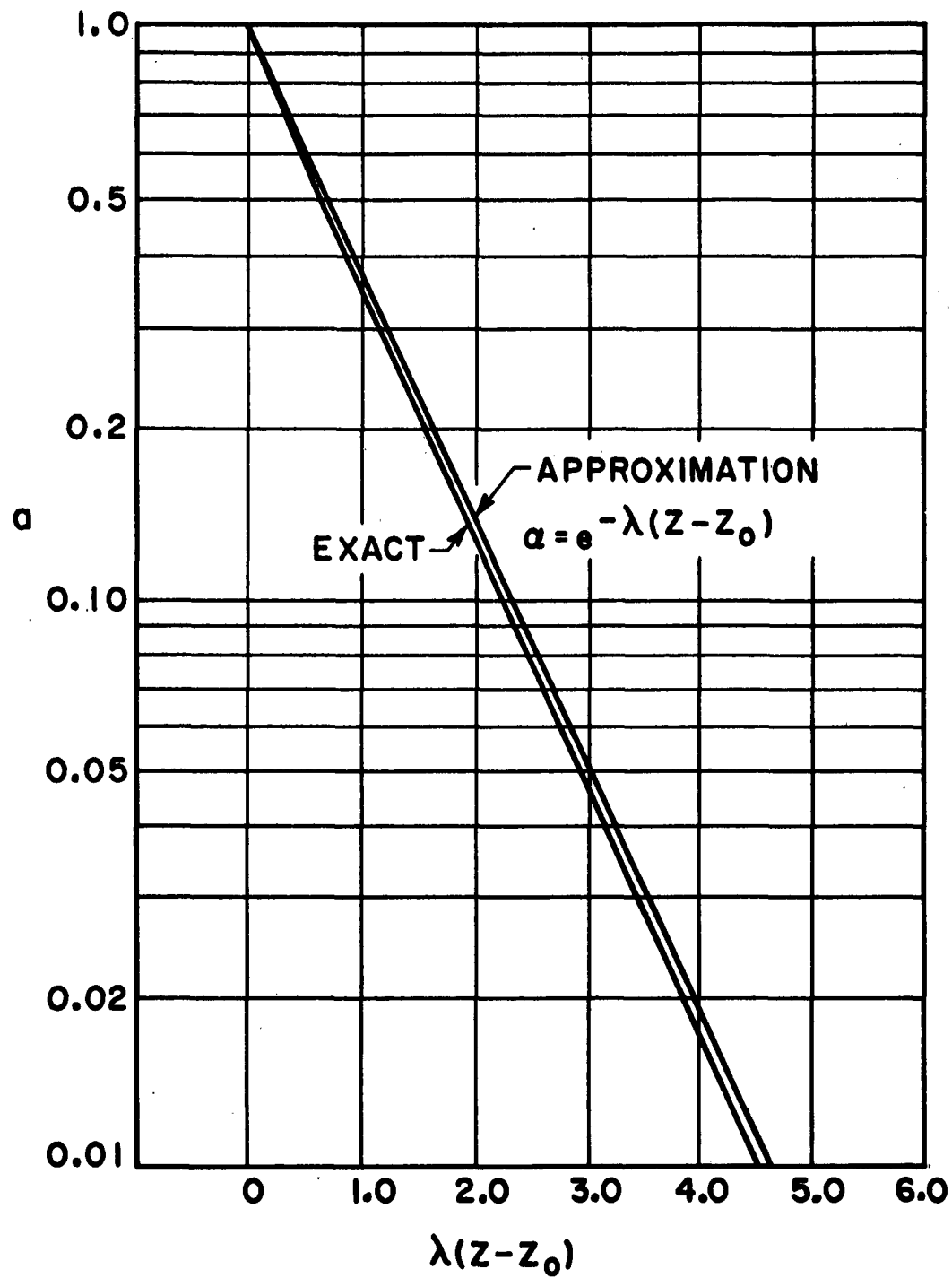
FIG. 6--Comparison of exact solution for α with exponential approximation

$$x_e = \left(\frac{2m_i}{m_e}\right)^{\frac{1}{2}} \frac{1}{\lambda} \left[[1 - (1 - \alpha^2)^{\frac{1}{2}}]^{\frac{1}{2}} - 1\right] \quad , \tag{68}$$

for the turning point at $x_e = 0$, and

$$x_i = \frac{m_e x_e}{m_i} \quad . \tag{69}$$

The outgoing trajectory is symmetrical with respect to the turning point (see Fig. 7).

From Eq. (56), for the current density J_x , we have

$$J_x = -\left(\frac{2e^2 n_0 A_0}{Mc^2}\right) \alpha(1 - \alpha^2)^{-\frac{1}{2}} > 0 \quad , \tag{70}$$

and the total current per unit length of plasma front, measured in the y-direction is

$$J_L = -\frac{2e^2 n_0 A_0}{Mc^2} \int_{z_0}^{\infty} \alpha(1 - \alpha^2)^{-\frac{1}{2}} \, dz \quad , \tag{71}$$

which is most easily integrated in terms of $0 \leq \alpha \leq 1$, by substituting for $dz/d\alpha$ from Eq. (56). This gives

$$4\pi J_L = [16\pi n_0 (m_i + m_e)W^2]^{\frac{1}{2}} = H(z_0) \quad , \tag{72}$$

nine-tenths of which flows near the surface, in the region where $0.141 \leq \alpha \leq 1$. This is identical to the expression for surface currents, if one were to assume complete separation of magnetic field and plasma.
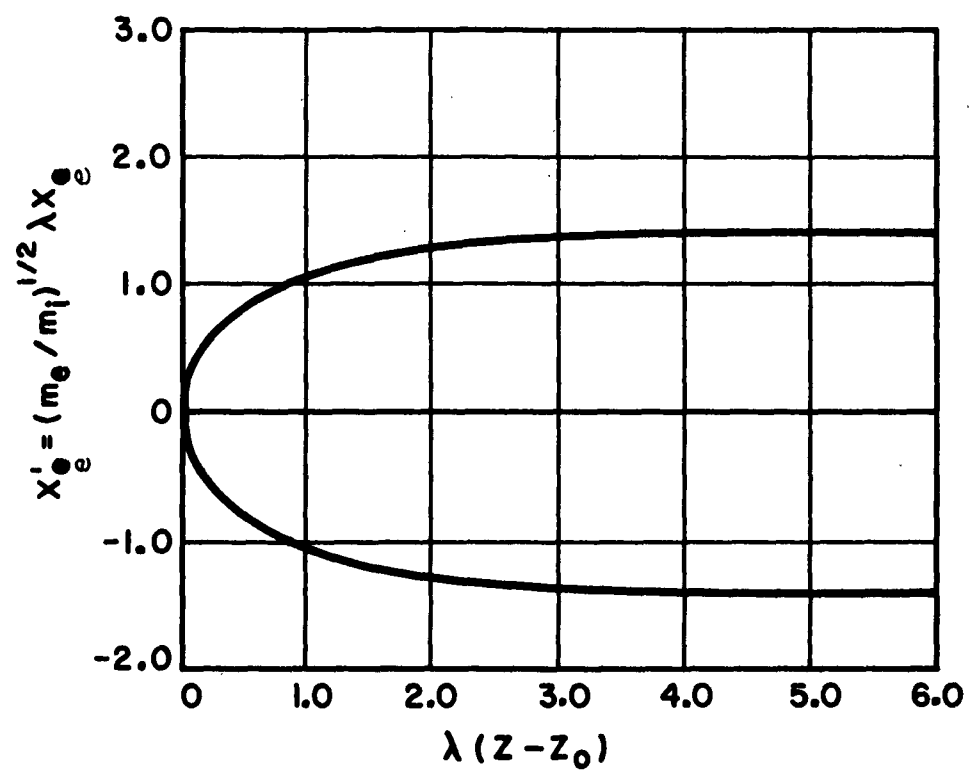
FIG. 7--Electron trajectory
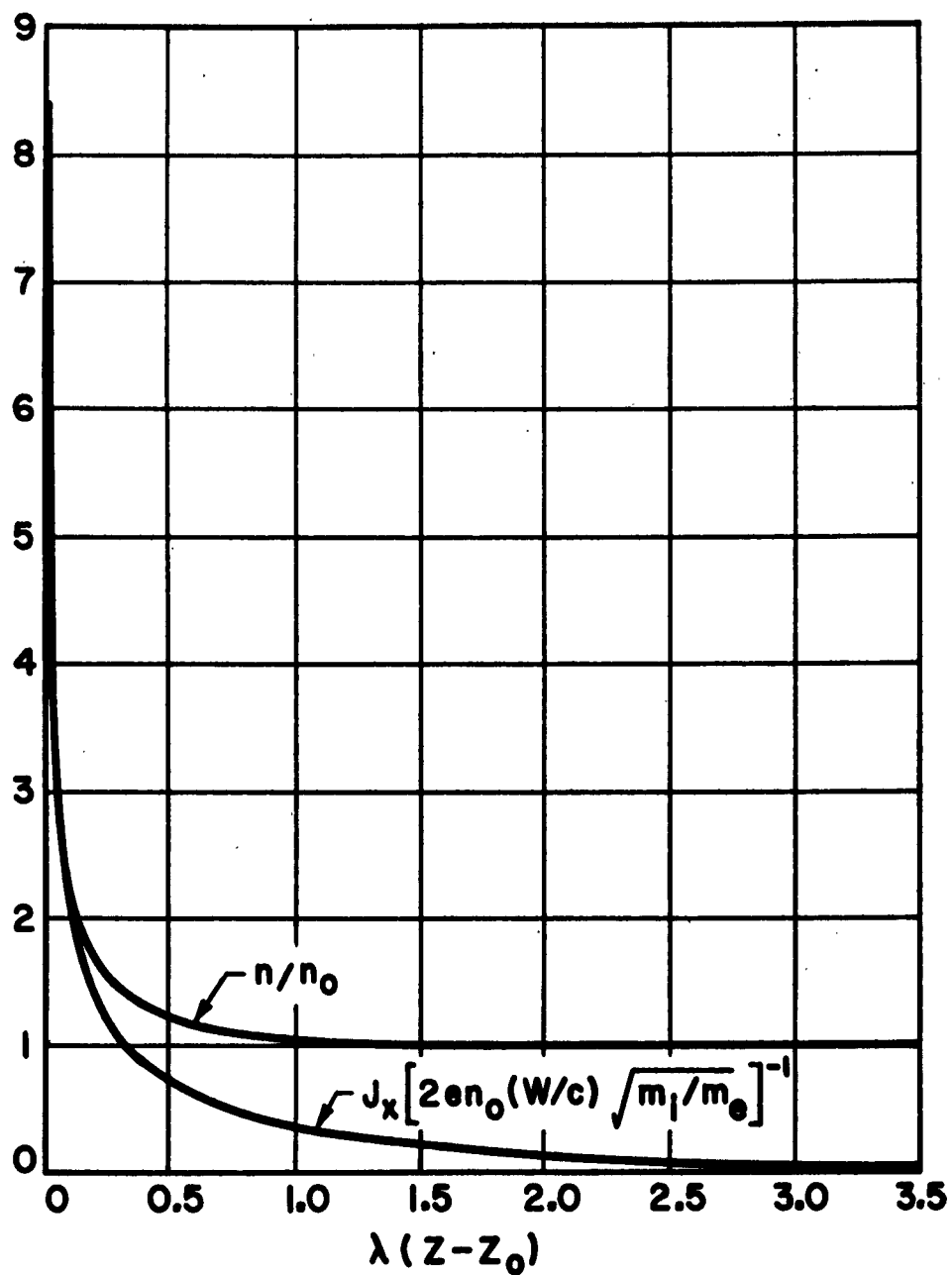
FIG. 8--Particle density n and current density, J_x , in the interplanetary plasma.

In order to verify the approximation of quasi-neutrality, we calculate the charge density from Poisson's Equation, which becomes

$$- d^2\phi/dz^2 = 4\pi\rho \doteq - (8\pi n_0 e W^2/Mc^2) \, [(m_i - m_e)(2 - 2\sqrt{1 - \alpha^2} + \alpha^2/\sqrt{1 - \alpha^2})$$

$$+ (m_i m_e)^{\frac{1}{2}} (U/W) \, \alpha/\sqrt{1 - \alpha^2} \,] \quad . \tag{73}$$

When this is integrated over $z_0 \leq z \leq \infty$ by means of the substitution $dz = (dz/d\alpha) \, d\alpha$, we find a net charge in the plasma per unit surface area of

$$Q \doteq - (m_i^2 \, c^2 n_0 / \, \pi m_e)^{\frac{1}{2}} \, (W/c)^2 \, [1 + (U/W)(m_e/m_i)^{\frac{1}{2}}] \quad , \tag{74}$$

approximately, to within $(m_e/m_i \ll 1)$.

To preserve overall neutrality, we require a positive surface charge of $(- Q)$ at $z = z_0$. This means that all electric lines of force which originate in the plasma end on the positive charges in the surface, so that there is no electric field in the region $z < z_0$, and the electric potential there is a constant. The surface charge corresponds to physical reality in that the more massive ions will penetrate further into the magnetic field than the electrons, while being retarded by the Coulomb force because of their slight separation from the electrons. We may verify the quasi-neutrality:

$$\left| \frac{n_i - n_e}{n} \right| = \left| \frac{\rho}{2en} \right| < \frac{m_i}{m_e} \left(\frac{W}{c} \right)^2 \left[3 + \sqrt{\frac{m_e}{m_i}} \left(\frac{U}{W} \right) \right] \quad . \tag{75}$$

neglecting the term in (U/W) gives

$$\left| \frac{n_i - n_e}{n} \right| < 0.005 \tag{76}$$

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at the surface of the plasma, and generally much smaller than 0.005 in its interior.

A more general discussion of this problem, including consideration of (one-dimensional) velocity distributions at $\infty = x$ is to be found in Grad's 1961 paper.

B. TWO-DIMENSIONAL MODEL

We now formulate an idealized problem, using the insights gained in section IV.A. We consider the two-dimensional problem first, i.e., a configuration of line currents, located at the origin of coordinates, and small in extent compared to the dimensions of the problem. We wish to find the magnetic field within a certain region containing the origin; the boundary of this region is to represent the interface between the GMF and the SCR. Within the region both the scalar (Ω) and vector (A_z) magnetic potentials must satisfy Laplace's equation:

$$\Delta \Omega = 0 = \Delta A_z \quad ; \quad \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 \quad , \tag{77}$$

with a __specified__ singularity at the origin due to the given configuration of line current sources.

In the preceding section we found that the magnetic field penetrates only a very short distance into the plasma stream; in our idealized model we assume that the field is completely excluded from the plasma by currents, J, per unit length, flowing in the interface. These currents must satisfy the relation

$$H = 4\pi J \quad , \tag{78}$$

where H is the total field at the boundary. Since normal components are continuous across the boundary we have the first boundary condition:

$$F = A_z - C = 0 \quad ; \quad C = \text{const.} \quad ,$$

which is equivalent to the requirement that the magnetic field be parallel to the interface. If C can be determined at one point on the boundary by considerations of symmetry, for example, and if the boundary were known, we should have a Dirichlet problem, which has a unique solution.

The boundary is not known; instead we have the second condition

$$H^2 = \beta^2 \cos^2 X \; ; \quad \beta^2 = 8\pi(2n_0 V^2)(m_i + m_e) \; ; \quad \beta > 0 \; ; \quad (79)$$

where X is the angle between the normal to the boundary and the x-axis which is now taken to be the direction parallel to the stream.

This second boundary condition must compensate for our lack of knowledge concerning the exact location of the bounding surface. We shall assume that the problem thus formulated is susceptible of a unique solution; however, this has not yet been rigorously established, to my knowledge, and would constitute a valuable addition to the theory of free-boundary problems.

We may express $\cos^2 X = (dy/ds)^2$, where $ds^2 = dx^2 + dy^2$ is the arc length along the boundary in the xy-plane (see Fig. 9). Thus, the boundary condition may be expressed as

$$- d\Omega = H \; ds = \pm \beta \; dy \; , \tag{80}$$

with $\pm$ depending upon whether the field lines are parallel or antiparallel to ds. Over some particular region of the interface we may then integrate the above equation:

$$G = \Omega \pm \beta y - K = 0 \tag{81}$$

$$K = \text{const.} \; .$$

Thus, the integral form of the two boundary conditions defines two one-parameter families of surfaces, F and G, the intersection of which is
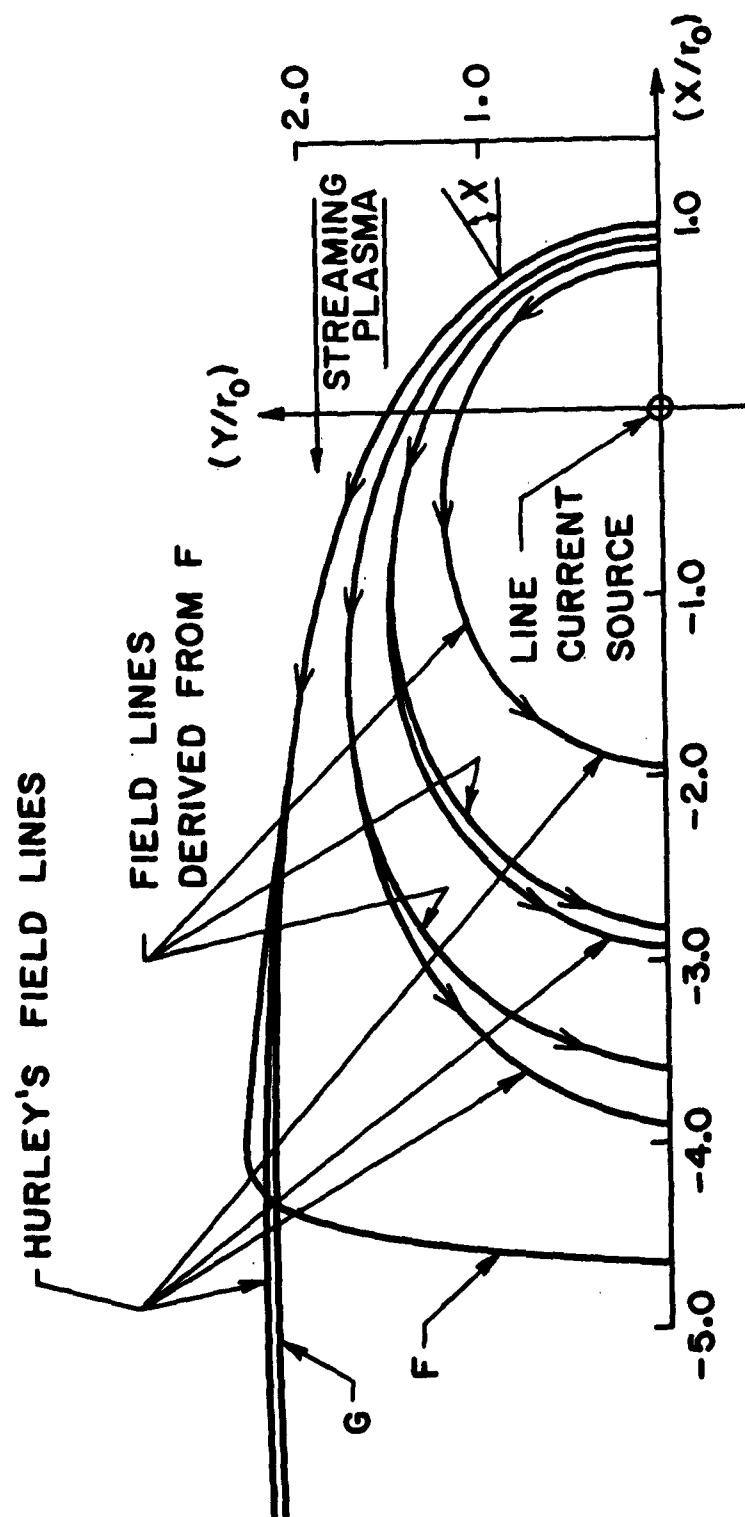
FIG. 9—The two-dimensional line current

the desired interface. In two dimensions Ω and A_z are Cauchy-Riemann conjugates, so that the choice of a representation for one automatically determines the other. One method of obtaining an approximate solution (see below) would be to represent Ω and A_z by an infinite harmonic series with undetermined coefficients plus separate terms to represent the source of the field. One could then evaluate the coefficients so that the two surfaces would be made to fit as closely as desired over the region of interest (co-location method). Again, it must be noted that the validity of the foregoing has not been rigorously determined.

Both Dungey (1961) and Zhigulev (1959c) have attempted to solve the problem of a two-dimensional magnetic line dipole in the streaming SCR, with the aid of various conformal transformations. However, the most lucid derivation is that of Hurley (1961a,b) who has treated both the problem of a line current transverse to the direction of the SCR, and the problem of the line dipole inclined at arbitrary angles to the SCR.

In the case of the line current, I, Hurley shows that the equation of the interface is

$$2 \cos (\beta y/4I) = \exp(\beta x/4I) \; ; \qquad (82)$$

the magnetic field may be derived from either the magnetostatic scalar potential

$$\Omega = I(y' + \arcsin U) \; , \quad x' = \beta x/4I \qquad (83)$$

$$U = (1 - \cosh x' \cos y')/(\cos y' - \cosh x') \; , \qquad (84)$$

or the vector potential

$$A_z = - I[x' + \log(2 \cosh x' - 2 \cos y')] \; . \qquad (85)$$

In Fig. 9 we have normalized to the smallest dimension,

$$r_0 = (4I/\beta) \log_e 2 \; . \qquad (86)$$

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By means of the co-location method described above one can obtain the series representation of the F and G surfaces to second-order approximations:

$$F \doteq \log_e(r/r_0) + 0.34473(r/r_0) \cos\theta + 0.01993(r/r_0)^2 \cos 2\theta - 0.36466 = 0 \tag{87}$$

$$G \doteq \theta - 1.03986(r/r_0) \sin\theta + 0.01993(r/r_0)^2 \sin 2\theta = 0 \tag{88}$$

where

$$r_0 = 2.7721/\beta \quad . \tag{89}$$

Figure 9 shows the F and G surfaces as well as Hurley's exact solution. Magnetic field lines inside the interface have also been plotted. It appears that substantial agreement between the F and G surfaces indicates a good approximation to the right answer; it may also prove possible to generate a quantitative estimate of the actual error based upon the difference between F and G.

Hurley finds $H' = 0.4H_p$, $H = H' + H_p$, at $(r_0, 0)$ which is comparable to the value $H' = H_p/2$ derived from the Ferraro-Dungey plane model.

The case of a line dipole inclined at angle α to the y-axis is more difficult, and the explicit expression for the fields or their potential is not derived by Hurley, although it is fully implicit in his solution. Thus, if

$$H_x - iH_y = H^* = d\Phi/dz \quad ; \quad \Phi = -\Omega + iA_z \quad , \tag{90}$$

then the nature of the singularity at the origin is such that

$$\Phi \to 2M/iz \quad \text{as} \quad z \to 0 \quad . \tag{91}$$

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Hurley transforms to the w-plane, such that the boundary is mapped onto a circle of radius a; i.e., $w = a e^{i\phi}$. By requiring that $w(z) \to z$ as $z \to 0$ he derives

$$\Phi = (2M/i) (1/w - w/a^2) ,\qquad (92)$$

for the field of a dipole bounded by a circle; he then finds the transformation $z = z(w)$ which distorts the boundary of the circle until the pressure condition is satisfied. Hurley then shows that for

$$a^2 = 16M/\beta\pi ; \quad M = \text{dipole moment} ,\qquad (93)$$

and

$$z' = z(\beta/\pi M)^{\frac{1}{2}} ,$$

the equation of the boundary is given in terms of ϕ as parameter:

$$\pi x' = - [1 - \sin(\phi + \alpha)] \log[\cos (\phi + \alpha)/(1 + \sin(\phi + \alpha))]^2 \quad (94)$$
$$+ \log[(1 + \cos \phi)/(1 + \sin(\phi + \alpha))]^2 + 2$$

$$y' = \begin{cases} - \sin(\phi + \alpha) + 2 - 2\alpha/\pi & \text{for } \pi > \phi > \pi/2 - \alpha \\ \sin(\phi + \alpha) - 2\alpha/\pi & \text{for } \pi/2 - \alpha > \phi > - \pi/2 - \alpha \\ - \sin(\phi + \alpha) - 2 - 2\alpha/\pi & \text{for } - \pi/2 - \alpha > \phi > - \pi . \end{cases} \qquad (95)$$

We have applied the co-location method to the case of $\alpha = \pi/2$, using the third-order approximation thus obtained as a trial surface in a relaxation calculation. To apply the relaxation method one considers the Dirichlet problem for the vector potential with a given trial surface. If the approximation is a reasonably good one, only relatively small perturbations of the boundary surface are required to obtain agreement with the second boundary condition. In this case the trial surfaces quickly converged to Hurley's solution.

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C. THREE-DIMENSIONAL MODEL

To the present time (July, 1962) there have been only a half-dozen published papers which give some quantitative estimate of the three-dimensional interface around a stationary magnetic dipole perpendicular to the flow of the SCR. The first to deal with this subject was Beard (1960) who represented the interface by a function $F(r, \theta, \phi)$ derived by an approximate method from the second (pressure) boundary condition expressed in the form of a nonlinear partial differential equation in F. The first (tangential) boundary condition is ignored entirely and replaced by the assumption that the original dipole field is only changed in magnitude by a constant, k, at the boundary, without any change in direction; the component of the field normal to the boundary is simply ignored. In fact, this is a fruitful source of error since the curvature of the boundary necessarily results in considerable distortion, as well as augmentation of the GMF. However, provided one assumes the appropriate value of k (Beard takes k = 2) this does constitute a surprisingly good approximation, at least in the two-dimensional case. Beard notes that near the poles one expects to find neutral points corresponding to Chapman's N (see Fig. 3) at which the direction of stream flow is locally parallel to the surface, so that $H = 0$; he also believes that in this region the curvature of the surface will be much higher than elsewhere, and may even require modified boundary conditions, since these are based on the assumption that the ion Larmor radius is small relative to the curvature of the surface.

Although the two-dimensional work indicates the usefulness of Beard's first supposition it is unlikely that the radius of curvature would ever approach the Lamor radius; if it did one would have almost a cusp in the surface, ideally placed to trap incoming particles, whose pressure would then broaden it. Such a region would be expected to be highly unstable in any case. (Northrop and Teller, 1960; Chang, 1962). Beard derives an expression for the shape of the surface in the earth-sun plane ($\partial F/\partial \phi = 0$) which he finds to be approximately circular. On the nightside of the surface, Beard estimates the length of the tail

(see Fig. 1) to be approximately

$$L = R_0 V/v_t , \tag{96}$$

where v_t is the thermal velocity, and R_0 is the maximum cross-section of the cavity, the tail being assumed to be approximately conical in shape. With an estimate of $R_0 < 1.8 r_0$, r_0 the minimum dimension of the cavity, approximately $r_0 = 10R$, and $T = 10^5 \, ^\circ K$, the tail would be a cone of altitude approximately $100R$.

Spreiter and Briggs (1962) claim to have discovered inaccuracies in Beard's work and have re-done it obtaining solutions in the equatorial and meridian plane. Unfortunately, being unaware of Dungey's work at the time, the authors used Ferraro's boundary condition

$$H^2/8\pi = nmV^2 .$$

This error was noted and rectified in a later letter, indicating that their original dimensionless lengths must be multiplied by $(2)^{-1/2}$ when restored to physical dimensions.

More recently Beard (1962) has published another paper on this subject in which he uses the first approximation to calculate surface currents in the interface, and integrates these numerically to find their effect on his assumed dipole field; the field is then altered and the calculation repeated, thus accounting in some measure for the distortion of the GMF due to curvature of the surface. This is only calculated for the meridian plane, which once again yields a circular shape, only 3% larger at the poles than at the equator.

Slutz (1962) and Midgeley and Davis (1962) have treated the case of a dipole enclosed by a uniform pressure (no $\cos^2 X$ in second boundary condition). Both solutions were numerical, but by different methods, yet the results for the cross-sections agree to within 1% at the pole, and 3% in the equatorial plane.

D. ANALOGIES FROM GASDYNAMICS

A common technique in the study of hypersonic gaseous flow around solid objects is to fix the object in a wind tunnel: one then observes a detached "standing" shock wave on the upstream side of the test object, the geometry and thickness of the shock wave depending upon Mach number, temperature, viscosity, shape of object, etc. Downstream of the object one may observe a "wake," an area of relatively low pressure, with a sudden change in fluid velocity at its boundary, accompanied by turbulence.

The gas in passing through the shock front is reduced to subsonic velocities, regaining speed as it flows around the object and its wake. There is a third region of importance, and that is the boundary layer between the object and the gas. This is a transition region between a region where no gas is allowed to flow, i.e., the object itself, and the region of irrotational flow; it is due to the viscosity of the fluid which requires that it be stationary at the surface of the object. The thickness of this boundary layer increases as one moves downstream and is proportional to the square root of viscosity for small viscosities.

The GMF-SCR interaction has been compared with just such a problem, since the protons are hypersonic: the GMF would be the test object, and the interface has been called the boundary layer. On the basis of these analogies it has been suggested that there should be another transition region, or shock wave, in the SCR between the earth and sun due to the presence of the geomagnetic obstacle to the flow, and also a "wake" on the night side. If these regions do exist they have not been detected.

There is another reason for doubting the existence of this "detached" shock wave: if we calculate values of density, velocity, temperature and magnetic field upstream of the presumed shock by means of continuity relations using values measured by Explorer X, we find a subsonic to supersonic transition for SCR passing through the shock. Such a condition is forbidden by the second law of thermodynamics in the case of both classical (Liepmann and Roshko, 1957, pp. 56-61), and hydromagnetic shocks (de Hoffmann and Teller, 1950, pp. 698-700). That is, although the conservation of energy allows subsonic to supersonic transitions, the requirement of increasing entropy will not permit increasing the average

velocity of a fluid at the expense of its thermal energy by passing it through a dissipative region like a shock wave.

Previously we have considered the SCR to be specularly reflected at the interface; every measurement has shown considerable turbulence at this boundary, and it may be more correct to assume diffuse reflection at the boundary. This would reduce dependence upon the angle of incidence, but not greatly alter the nature of the problem. A more extreme assumption would be that particles can actually penetrate the transition region which separates two fairly stable regions, the magnetosphere and the SCR, and are not reflected at all. Cahill and Amazeen (1962) find the thickness of the boundary to be 100 to 1000 km near the subsolar point, i.e., still relatively thin. Under this assumption one could apply the nonrelativistic relations of de Hoffmann and Teller (1950) for a hydro-magnetic shock, since they can be derived by integrating the classic continuity relations across the transition region between two equilibrium states.

For a normal, one-dimensional shock in a transverse magnetic field, these relations are:

$$m n u = A \quad ; \quad H u = B \quad , \qquad (97)$$

(A, B, C, D are constants) and

$$p + mnu^2 + H^2/8\pi = C \quad ; \quad E + u^2/2 + p/nm + H^2/4\pi nm = D \quad , \quad (98)$$

where m = particle mass, E = internal (thermal) energy per unit mass and p = nkt , pressure. For an ideal gas we also know that $E = (3/2)nkT/nm = 3kT/2m = (3/2)p/nm$. The subscripts $(_1)$ or $(_2)$ are understood since the above equations apply on either side of the transition region (see Fig. 10).

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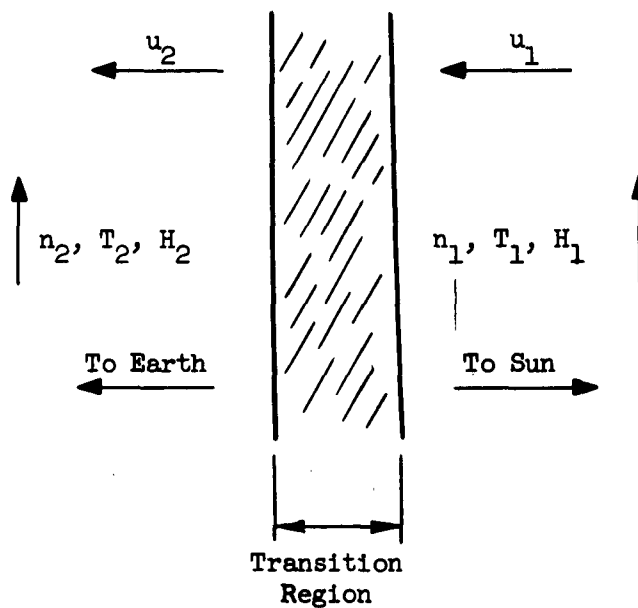
FIG. 10--Penetration Model.

The constants A, B, C, D can be evaluated if four of the eight un-known quantities, u, n, T, H are specified. In this case let us assume the quantities upstream of the transition region, or interface, to be given by the Explorer X results:

$$u_1 = 300 \text{ km/sec} \quad ; \quad n_1 = 10/\text{cm}^3 \quad ; \quad H_1 = 10\gamma \quad ;$$

$$T_1 = 0.5 \times 10^6 \text{ }^\circ K \quad .$$

thus, if we substitute for n, T, and H, the last equation becomes a cubic in u:

$$4 A u^3 - 5C u^2 + 2 DA u + B^2/8\pi = 0 \quad . \tag{99}$$

By symmetry it is clear that $u = u_1$ is a root of the above equation; upon dividing by $(u - u_1)$ we obtain the quadratic:

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$$4 \, A \, u^2 + (4 \, Au_1 - 5C)u + (2 \, DA + 4 \, Au_1^2 - 5Cu_1) = 0 \, , \qquad (100)$$

of which only the positive root is admissible,

$$u_2 = 108 \text{ km/sec} \, ,$$

and, therefore,

$$n_2 = 27.8 \quad ; \quad H_2 = 27.8\gamma; \ T_2 = 2 \times 10^6 \, ^\circ K \, , \quad p_2 = .76 \times 10^{-8} \text{ dy/cm}^2 \, ,$$

The value of H_2 agrees with results of Explorer X and Pioneer I which are approximately 30γ. However, it should be noted that the data is sparse in the case of Pioneer I and in the case of Explorer X we really have an oblique "shock" at some unknown angle to the direction of the flow. Furthermore, in preliminary results from Explorer XII it was mentioned that $H_2 \doteq 60\gamma$, presumably measured at around local noon. The value of H_1 was not given; however, it was stated that "the location of the outer edge of the region of trapped particles appears to be at or below the termination of the geomagnetic field, (IGY Bull., 1962b, p. 3). The low-energy proton analyzer with a threshold of 200ev, ceiling of 20 kev , (IGY Bull., 1961b), measured no protons below the transition region. However, the stream energy $mu_2^2/2 \doteq 55$ ev, and the thermal energy corresponds to around 260 ev , so that measurements neither refute nor substantiate the foregoing calculation. Since the speed of a compressional wave propagating normal to the magnetic field is

$$c^2 = 5p/3nm + H^2/4\pi nm \, , \qquad (101)$$

we have

$$c_1 = 1.035 \times 10^7 \quad ; \quad c_2 = 2.02 \times 10^7 \text{ cm/sec} \, ,$$

with Mach numbers, respectively,

$$M_1 = u_1/c_1 = 2.9 \quad ; \quad M_2 = u_2/c_2 = 0.535 \quad .$$

This standing-shock model might explain how particles penetrate the magnetosphere, and perhaps account for the experimental measures of the cavity dimensions; the amount of mass collected by a cross-section of $\pi(10R)^2$ is 2×10^{-7} earth masses/10^9 years . If the GMF penetrates all space, and is only altered near the interface due to surface currents, then the induced field H' of these currents is given by

$$H' = (H_2 - H_1)/2 = 8.9\gamma \quad , \tag{102}$$

and

$$GMF = (H_2 + H_1)/2 \quad ; \quad GMF = 18.9\gamma \quad . \tag{103}$$

The earth's magnetic dipole field will have this value when $r = 12R$, which is closer to the experimental results than the Dungey-Ferraro estimate of $8.1R$.

It should be noted that this concept of the steady-state standing shock wave is different from that of Singer (1957) who believes that a burst of SCR leaving the sun gives rise to a shock wave in much the same manner that a "driver" gas causes a shock wave in a laboratory shock tube. It is this traveling shock wave which causes the first phase of the geo-magnetic storm upon striking the earth, according to Singer.

The calculations of this section make no pretense to accuracy, but were intended primarily as a vehicle for discussion of penetration and shock wave models. The reader is also referred to recent articles by Axford (1962) and Kellogg (1962).

E. STABILITY

Experimental data from all rocket probes so far have indicated a considerable amount of turbulence in the region of the interface; Explorer X noted close inverse correlation with fluctuations in plasma density. Even if the interface is stable, the fluctuations in the SCR will move it back and forth; however, it has long been believed, on the basis of other geophysical phenomena, that the impact of sudden fluctuations generates unstable surface waves in the interface. Parker (1958) considered the magnetic field to be embedded in an infinitely conducting, incompressible, inviscid plasma, the plane surface of which is subjected to a wind at various angles. The simple application of linearized hydromagnetic theory yielded a complex dispersion relation for waves in the surface, and convinced Parker that instability was possible.

Hurley (1961c, p. 42) has generalized Parker's approach for different orientations of magnetic field relative to the impinging wind and finds that most cases of interest are, in fact, stable. However, in the noon meridian plane, for example, instability sets in as one moves toward the poles past a certain critical latitude θ_c given by (assuming the interface approximately circular in this plane)

$$\tan \theta_c \doteq 2/\epsilon \quad ; \quad \epsilon^2 = \rho/nm \quad , \tag{104}$$

where ρ is the density of the plasma below the interface and nm is the density of the SCR. If we estimate $\epsilon \doteq 1$, then $\theta_c \doteq 63.5°$ geomagnetic latitude, close to the auroral zones. The curvature of the actual field would add to the stability.

The calculations of Parker and Hurley, however idealized, provide our only quantitative basis so far for speculations concerning stability of the GMF-SCR interface.

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